Key

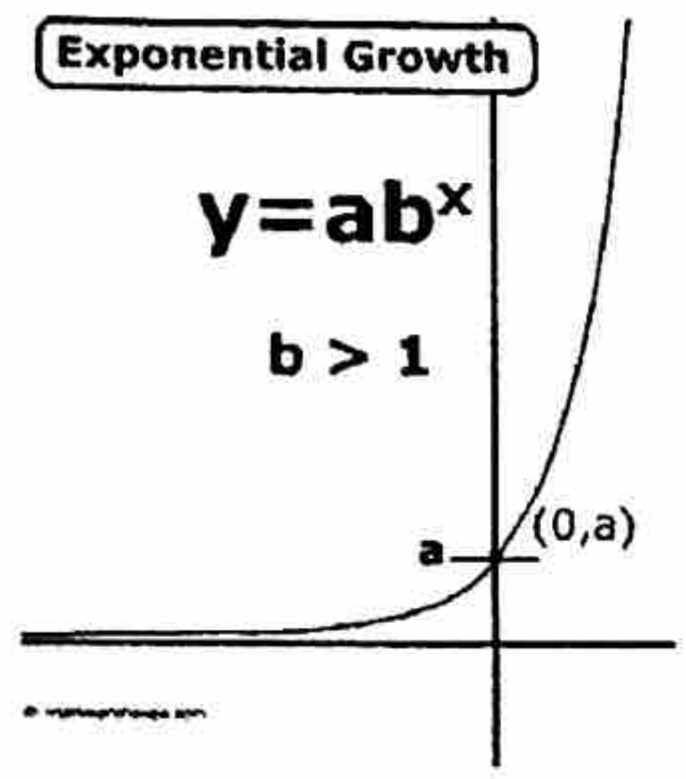
Lesson #45
The Exponential Function

EQ #2 What are the key differences between linear and exponential functions (graph/table/equation)?

An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. An exponential function is not linear.

* variable is the exponent.

Exponential Growth

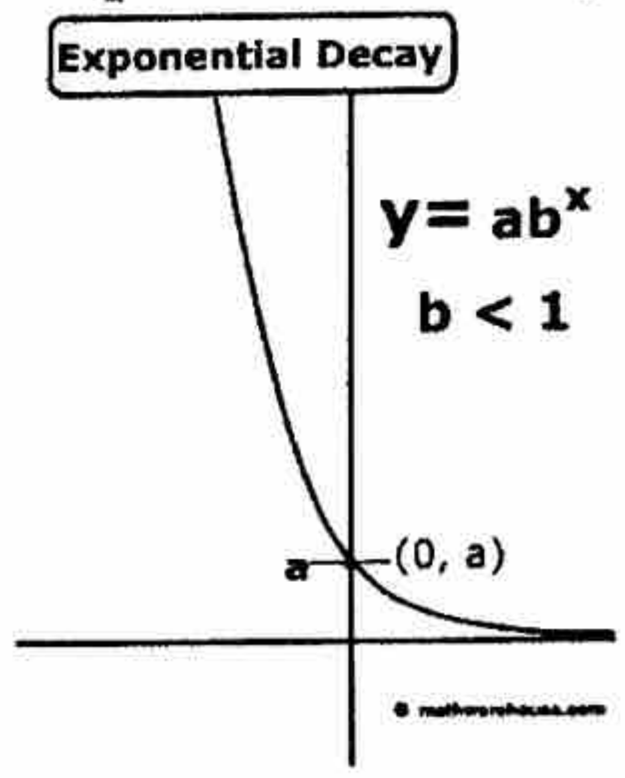


Domain:

Range:

Asymptote:

Exponential Decay



<u>**Asymptote**</u>: You will notice that the graph of exponential functions get closer and closer to the *x*-axis, but never actually *touch* it. This imaginary line (for now will only be the *x*-axis) is called an ***asymptote.***

Linear Function (constant rate) **Vs.** **Exponential Function (changing rate)**

$y = 3x + 2$

x	-2	-1	0	1	2
y	-4	-1	2	5	8

3 3 3 3

As the x values increase by 1, the y values increase by 3.

$y = 2 \cdot 3^x$

x	-2	-1	0	1	2
y	2/9	2/3	2	6	18

*3 *3 *3 *3

As the x values increase by 1, the y values multiply by 3.

<u>**Example 1**</u>**: Determine whether the function represents *exponential growth* or *exponential decay, or neither.* EXPLAIN why it is growth or decay. Check on the calculator.**

1. $y = x^2 + 3x$

Neither
X is not the exponent.

2. $y = \left(\frac{5}{4}\right)^x$

exponential growth $b > 1$

3. $y = 2 \cdot (6)^x$

exponential growth $b > 1$

4.

x	1	2	3	4
y	3	18	108	648

*6 *6 *6
exponential growth
multiply by 6.

5. $y = 3 \cdot (0.7)^x$

exponential decay
$b < 1$

6. $y = 5 \cdot \left(\frac{1}{3}\right)^x$

exponential decay
$b < 1$

7.

x	1	2	3	4
y	-5	-16	-27	-38

-11 -11 -11

Neither

8. $y = \frac{1}{2} \cdot (3)^x$

Example 2: **Evaluate the function for the given value of x.**

1. $y=4^x; x=3$

$y=4^3=4.4.4=$ ~~48~~ 64

2. $y=-2(5)^x; x=3$

$y=-2(5)^3$
$=-2(125)$
$=-250$

on your own

~~Homework~~: Complete the problems below.

Determine whether the table or equation represents an exponential function. EXPLAIN.

1.

x	1	2	3	4
y	2	8	32	128

$*4$ $*4$ $*4$

growth $*4$ every time

2.

x	0	1	2	3
y	6	9	18	33

$x1.5$ $*2$

Neither, not a constant multiple

3. $y=4\cdot 5^x$

growth because $b>1$

4. $y=12\cdot x^{②}$

Neither, x is not the exponent

5. $y=7x+3$

Neither, linear equation

6. $y=4\cdot\left(\frac{3}{2}\right)^x$

growth $\frac{3}{2}>1$

Determine whether the function represents *exponential growth* or *exponential decay*. EXPLAIN.

7. $y=(2)^x$

growth $b>1$

8. $y=\left(\frac{1}{8}\right)^x$

decay $b<1$

9. $y=.25\cdot\left(\frac{6}{5}\right)^x$

growth $\frac{6}{5}>1$

10. $y=5\cdot(7)^x$

growth $7>1$

11. $y=5\cdot\left(\frac{3}{4}\right)^x$

decay $\frac{3}{4}<1$

12. $y=\left(\frac{9}{4}\right)^x$

$\frac{9}{4}>1$ growth

Evaluate the function for the given value of x.

13. $y=(-2)^x; x=4$

$y=(-2)^4$
$y=16$

14. $y=2(3)^x; x=5$

$y=2(3)^5$
$y=486$

Key

Lesson #46
Exponential Growth and Decay

EQ #2 What are the key differences between linear and exponential functions (graph/table/equation)?

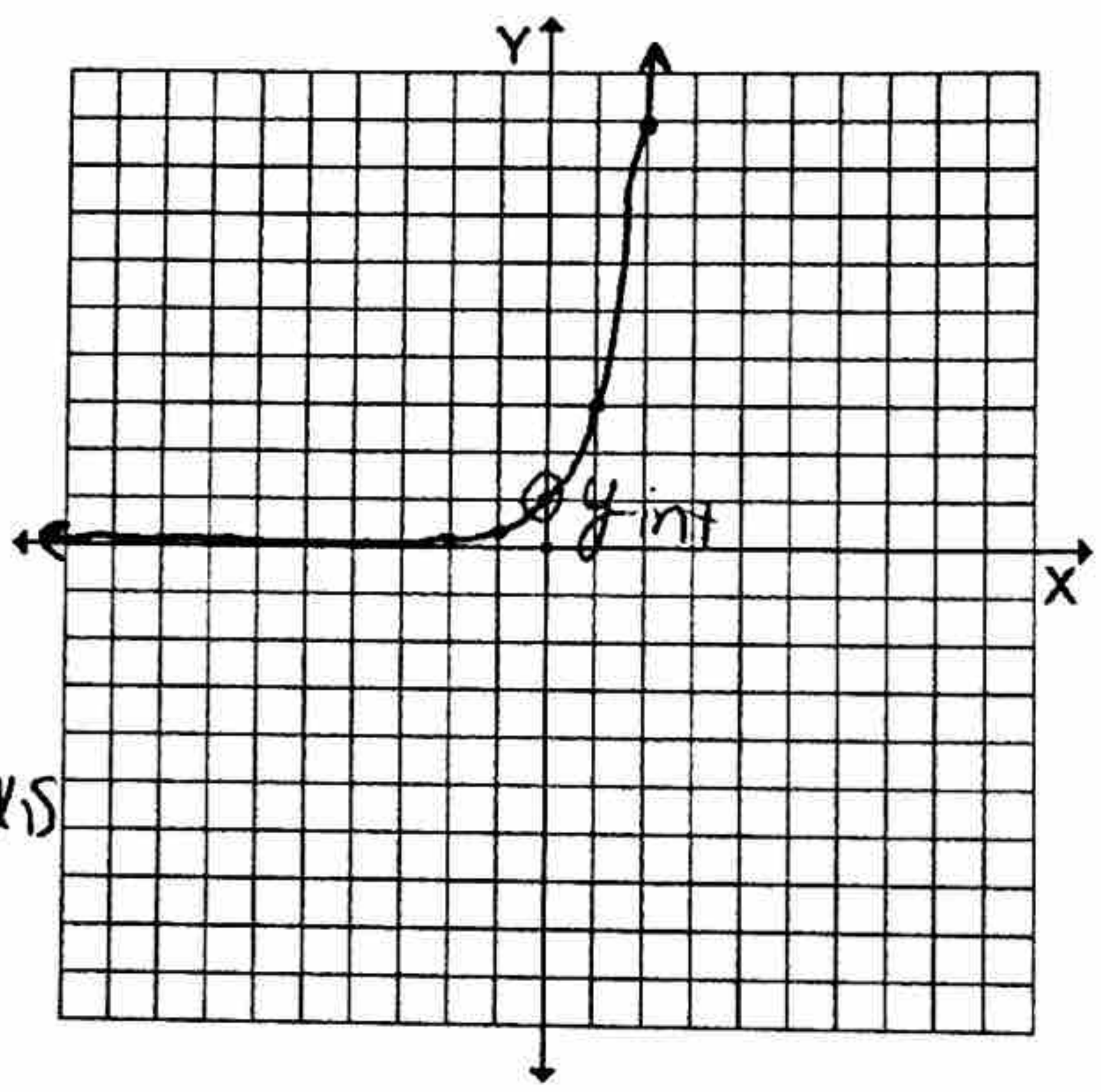
exponent is a variable

Exponential Functions: $y = a \cdot b^x$	
Initial Amount (starting point): $\underline{a}$ y-intercept	**Given point or time:** x
Growth or Decay Rate: $b - 1$ **(change to a percent)**	**Amount at any given point:** y

$y = 1 \cdot 3^x$

Example 1: **Graph** $y = 3^x$. **Use the chart below to help.**

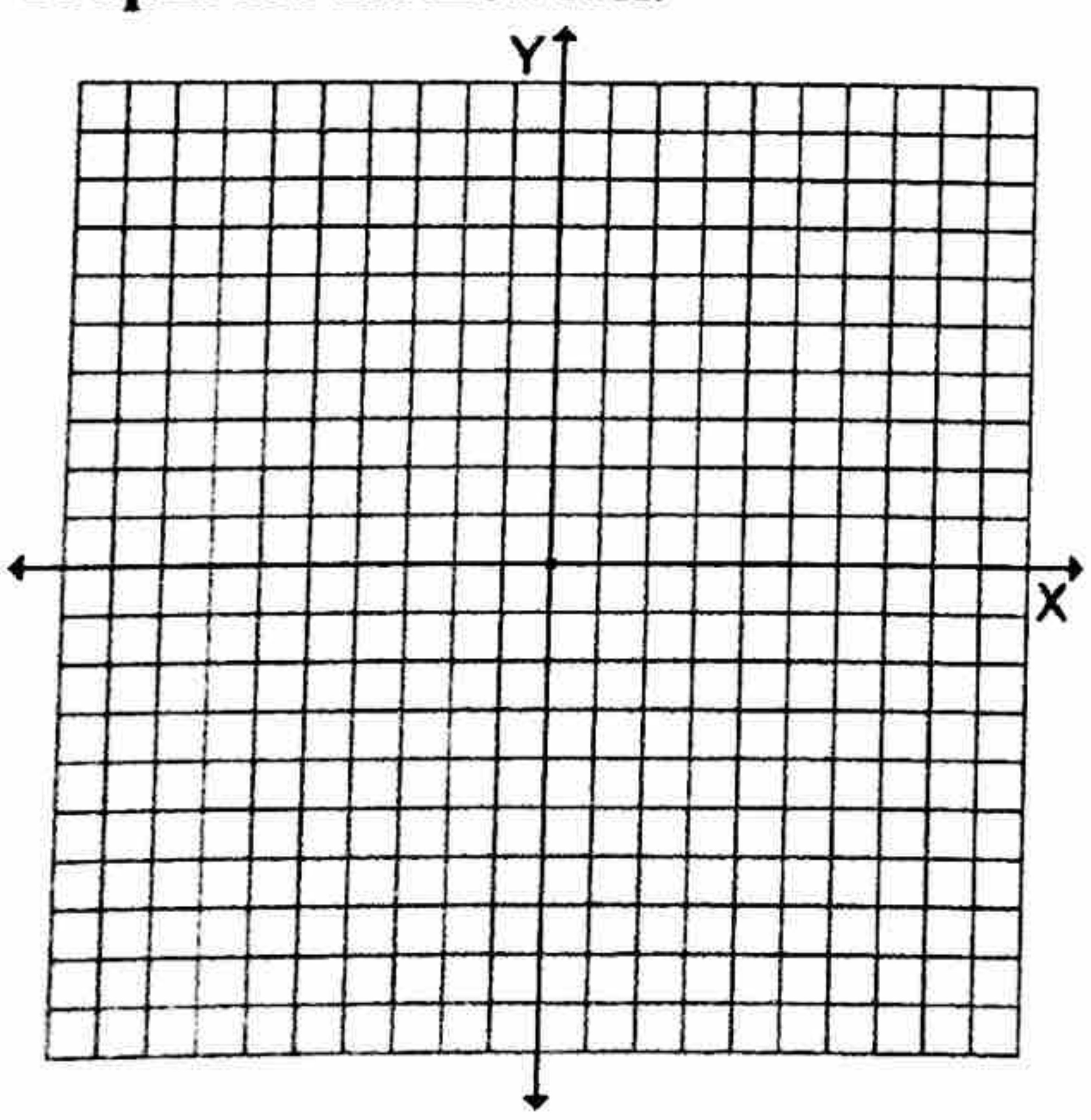
x	-2	-1	0	1	2	3
$y = 3^x$	$\frac{1}{3^2} = \frac{1}{9}$ .111	$\frac{1}{3} = .333$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$



- Is the graph increasing or decreasing?
  Increasing
- Is that exponential growth or decay?
  growth
- What is the growth or decay rate (as a percent)?
  $3 - 1 = 2 * 100 = 200\%$
- What is the y-intercept of the graph?
  (0, 1)
- What do you notice about the graph as the x-values get smaller and smaller?
  The graph gets closer to the x-axis
- What is the x-intercept of the graph?
  None
- What is the asymptote? ← line graph aproaches
  x-axis
- What is the domain (x-values)?
  all real #s $\mathbb{R}$
- What is the range (y-values)?
  $y > 0$

Extra Graphs for Homework:

21.



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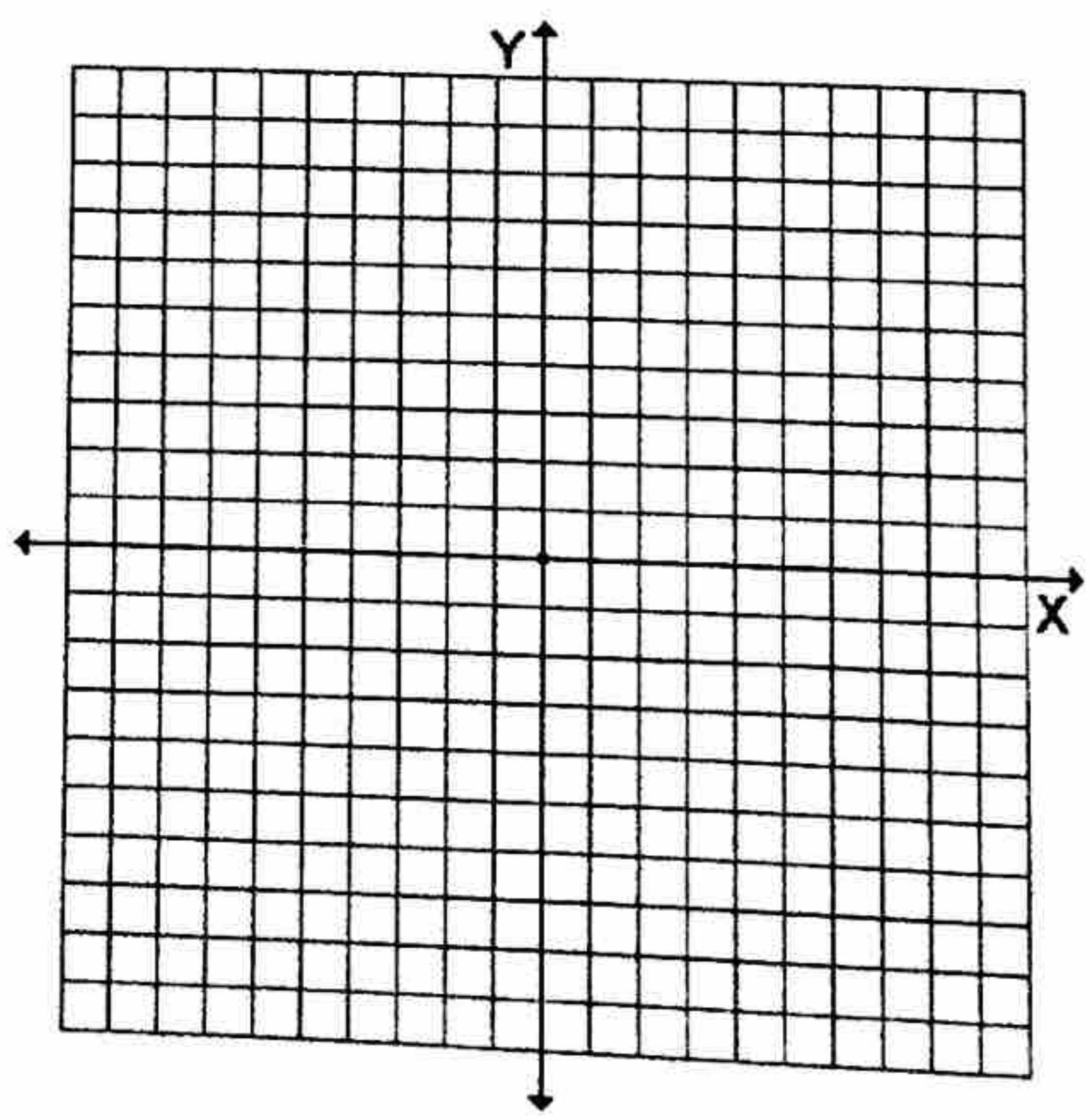
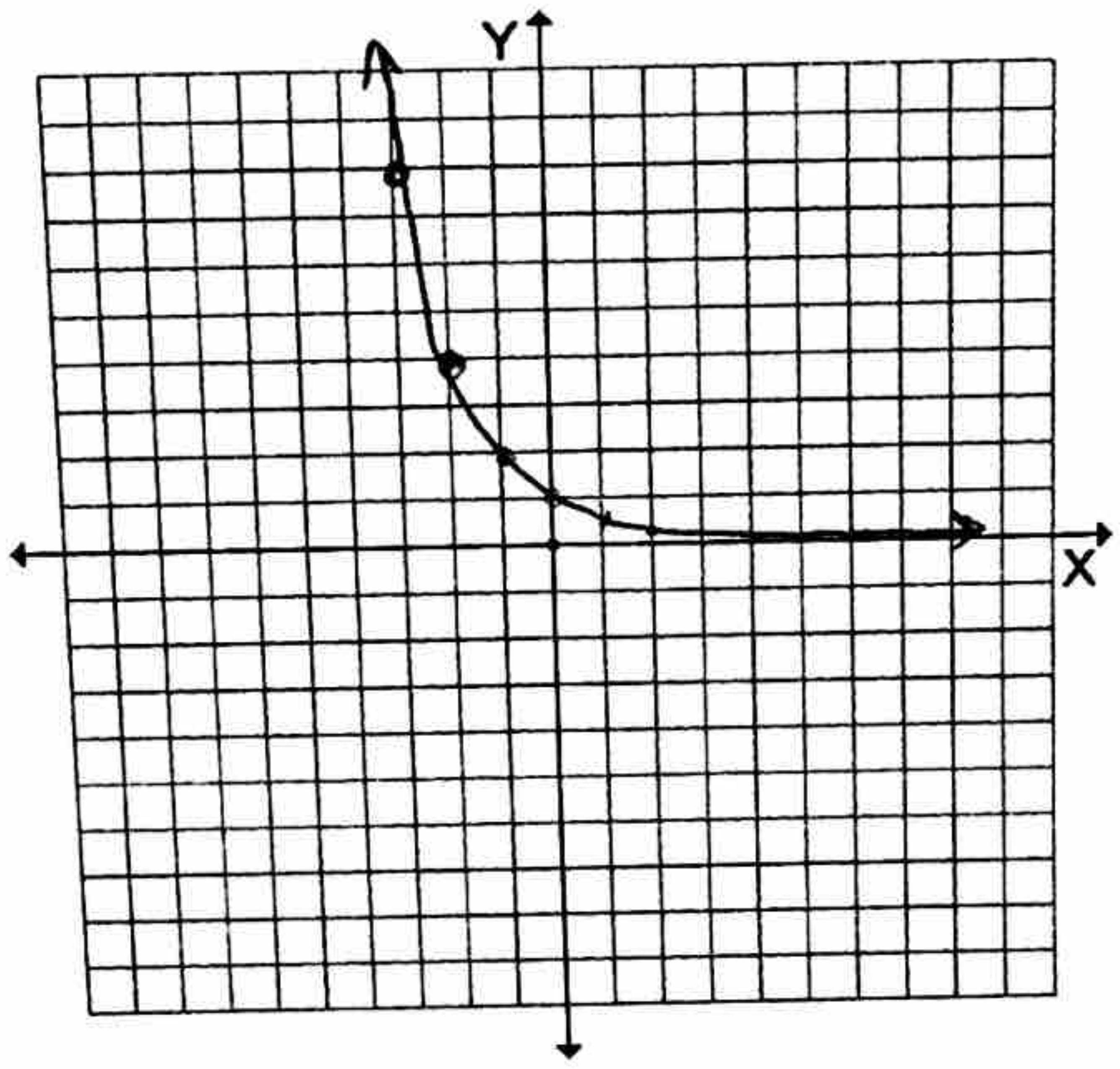


Example 2: Graph $y = (.5)^x$. Use the chart below to help.

x	-3	-2	-1	0	1	2	3
$(.5)^x$	8	4	2	1	.5	.25	.1

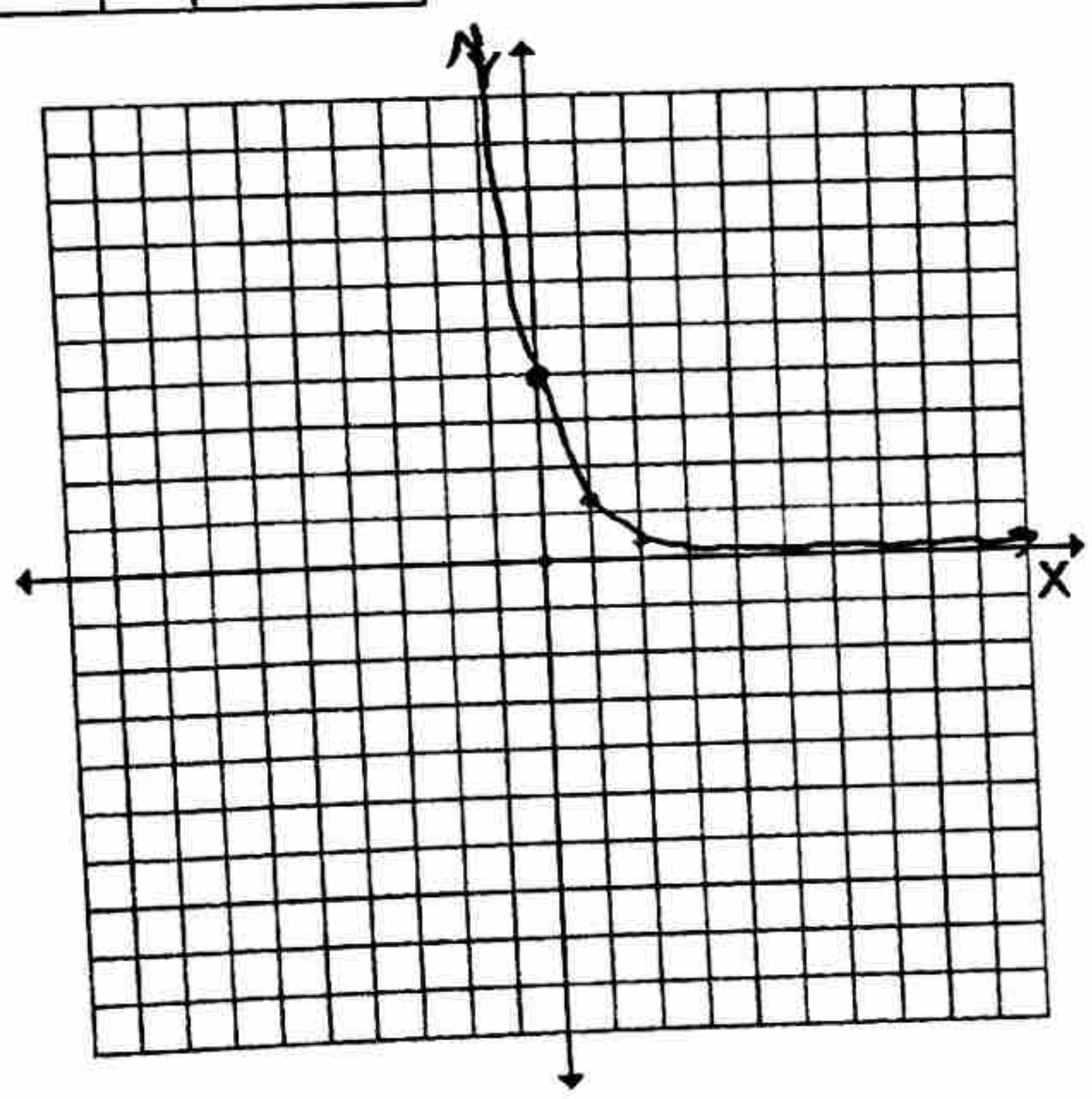


- Is the graph increasing or decreasing?
  decreasing
- Is that exponential growth or decay?
  decay
- What is the growth or decay rate (as a percent)?
  $.5 - 1 = -.5 * 100 = -50\%$
- What is the y-intercept of the graph?
  1
- What do you notice about the graph as the x-values get smaller and smaller?
  y-values get larger
- What is the x-intercept of the graph? None
- What is the asymptote? x-axis
- What is the domain (x-values)? all real numbers
- What is the range (y-values)? All positive numbers

On your own.

Example 3: Graph $y = 4(.3)^x$. Use the chart below to help.

x	-2	-1	0	1	2	3
$4(.3)^x$	44.4	13.3	4	1.2	.4	.1



- Is the graph increasing or decreasing?
  decreasing
- Is that exponential growth or decay?
  decay
- What is the growth or decay rate (as a percent)?
  $.3 - 1 = -.7 * 100 = -70\%$
- What is the y-intercept of the graph?
  (0, 4)
- What is the domain (x-values)?
  all real numbers
- What is the range (y-values)?
  all positive numbers.

Lesson #47
Start Thinking

Jimmy's grandfather has agreed to sell his car to Jimmy at whatever time he has saved up enough to buy it. His grandfather also agreed to sell the car at whatever the "low end" quoted price is on an automobile value website when Jimmy is ready to buy it. According to the website, the current "low end' value of the car is $11,000, and this value will decrease by 5% each month. Currently, Jimmy has $2400 saved and is planning on saving $350 more per month. After how many months will Jimmy be able to buy the car?

a. Is the value of the car each month an exponential or linear function? What is the rate?

Exponential ~~.05~~ -5%

b. Write a model (equation) for the value of the car. $y_1 =$

$y = 11000(1-.05)^x = y = 11000(.95)^x$

c. Is the amount of money Jimmy saves each month an exponential or linear function? What is the rate?

Linear $350 per month

d. Write a model (equation) for the amount Jimmy has saved. $y_2 =$

$y = 350x + 2400$

e. Fill in the table for each month with the value of car and the amount Jimmy has saved. Stop when Jimmy has enough money to buy the car.

x = number of months	y_1 = value of the car	y_2 = amount Jimmy has saved
0	11000	2400
1	10450	2750
2	9927.5	3100
3	9431.1	3450
4	8959.6	3800
5	8511.6	4150
6	8086	4500
7	7681.7	4850
8	7297.6	5200
9	6932.7	5550
10	6586.1	5900
11	6256.8	6250
12	5944	★ 6600
13	5646.8	6950

f. How many months will it take Jimmy to have enough money? ~~Is it a discreet or continuous domain?~~

12 months

g. If he buys it in the month he has enough money, how much money will he have left over?

6600-5944 = $656

Lesson #47
Exponential Growth & Decay

EQ #3 How can I model and solve a real-world problem using exponential growth or decay?

Exponential Functions: $y = a \cdot b^x$	
Initial Amount (starting point): a	**Given point or time:** x
Growth or Decay Rate: $b - 1$ **(change to a percent)**	**Amount at any given point:** y

Example 1: Writing Exponential Growth/Decay Equations

*** You must include units** *** Always round <u>final</u> answer to the nearest hundredth (2 decimals)**

1. You deposit $500 into an account that pays 8% annual interest. Write a model that can be used to represent the given situation. What is the account balance after 6 years?

$a = 500 \quad r = .08$

$y = 500(1 + .08)^x$

$y = 500(1.08)^6$

$y = 793.44$ dollars

2. Currently, the population of a certain city is 670,000. Each year the population increases by 2.3%. Write a model that can be used to represent the given situation. Estimate the population in 15 years.

$y = 670{,}000(1 + .023)^x$

$y = 670{,}000(1.023)^x$

$y = 670{,}000(1.023)^{15}$

$y = 942{,}343.6$ people in 15 years.

3. You bought a used car for $18,000. The value of the car will depreciate each year by 12%. Write a model that can be used to represent the given situation. Estimate the cost of the car in 5 years.

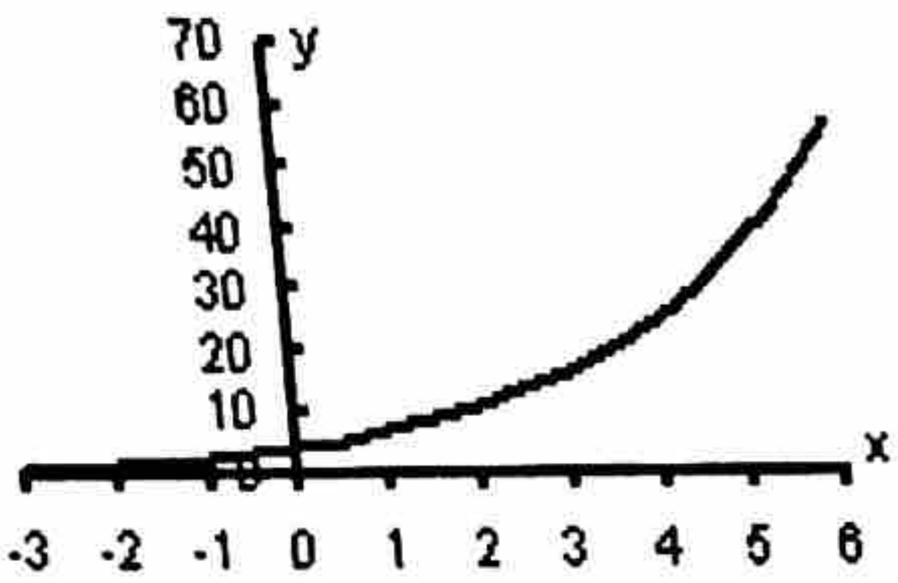
$y = 18000(1 - .12)^x$

$y = 18000(.88)^5$

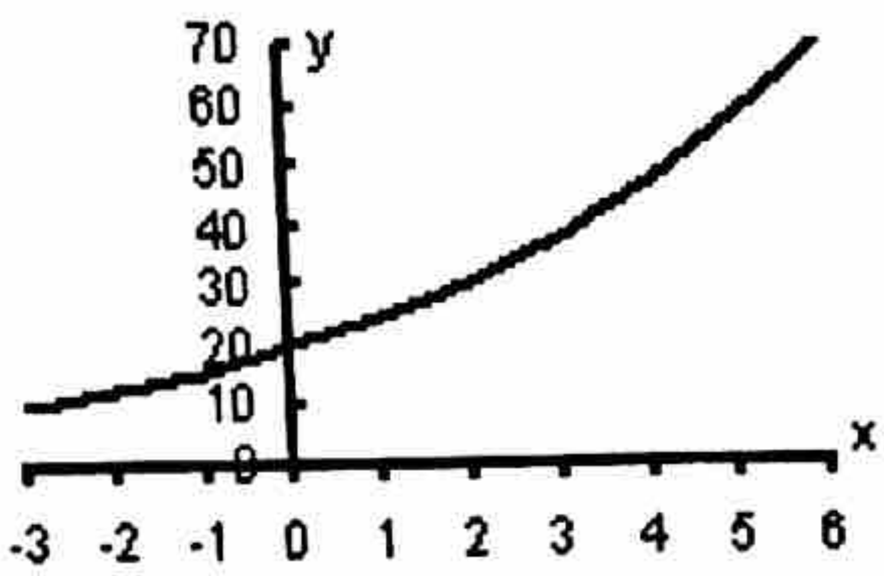
$y = 9499.2$ dollars

Example 2: Match each graph with the corresponding equation (place the letters in the blanks)

I.



II.



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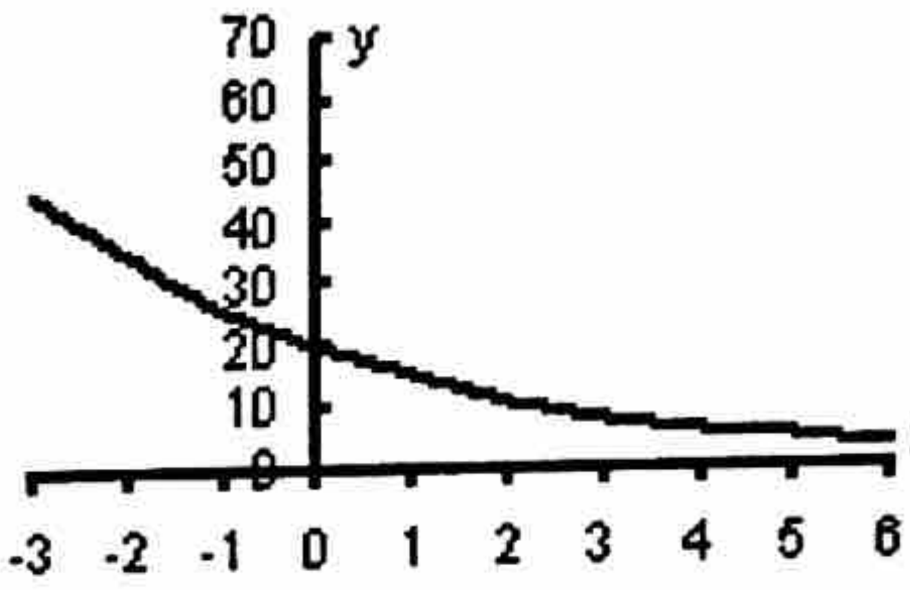

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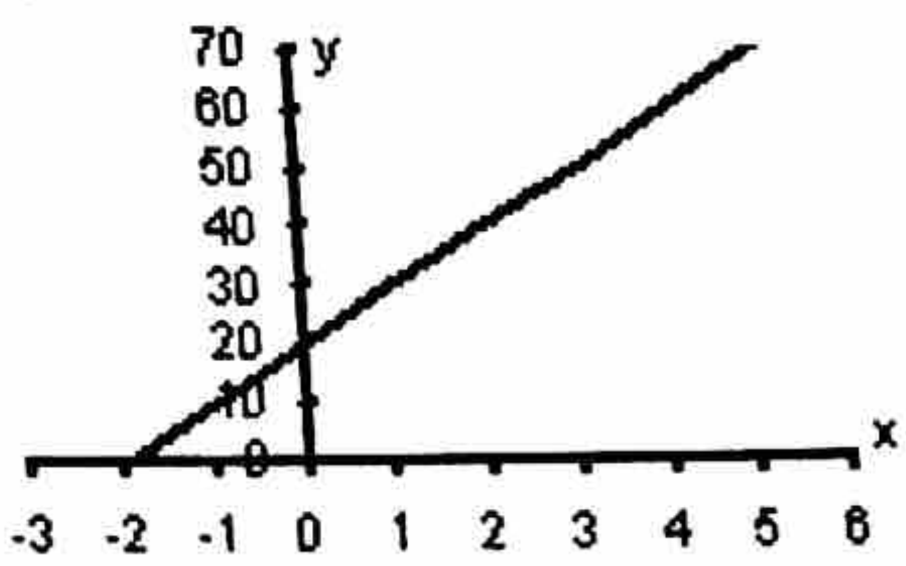

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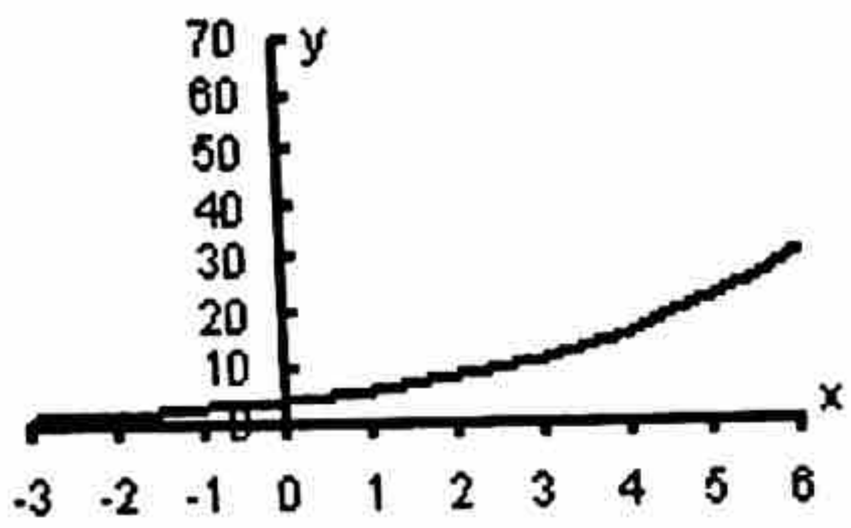
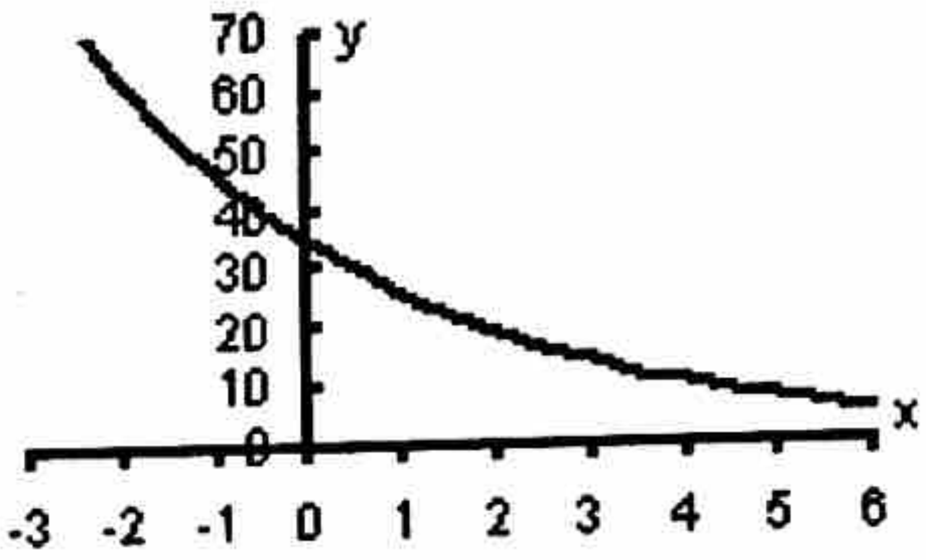


VI.



A. $y = 20(1.5)^t$ growth

B. $y = 5(1.25)^t$ growth

C. $y = 5(1.42)^t$ growth

D. $\cancel{y = 35(.7)^t}$ decay

~~E.~~ $\cancel{y = 20(.6)^t}$ decay

~~F.~~ $\cancel{y = 10x + 20}$ Linear

I. C II. A III. E IV. F V. B VI. D

Practice:

1. A savings certificate of \$1275 pays 6.5% annual interest. Find a model to represent the situation and then find the balance of the certificate in 15 years.

$y = 1275(1+.065)^x$

$y = 1275(1.065)^x$

$y = 1275(1.065)^{15}$ $y\ \$3279.1$

2. You bought a computer for \$1800. It depreciates by 29% each year. Find a model to represent the situation. Then find how much the computer is worth in 3 years.

$y = 1800(\cancel{x} - .29)^x$

$y = 1800(1-.29)^3$

$y = \$644.24$