4.6 Arithmetic Sequences

Learning Target:

I can identify patterns of arithmetic sequences and geometric sequences.

Success Criteria:

- I can write the terms of arithmetic sequences
- I can graph arithmetic sequences
- I can write arithmetic sequences as functions

Sequence: an ordered list of numbers, called terms; notation- a_n.

Each term has a specific position *n* in the sequence.

$$\begin{array}{cccc} a_1 & a_2 & a_3 & \\ 4, & 8, & 12, & \dots, a_n \end{array}$$

Arithmetic sequence: the difference between each pair of consecutive terms is the same. This is known as the common difference. Each term can be found by adding the common difference to the previous term.

I can write the terms of arithmetic sequences

Example 1: Write the next three terms of the arithmetic sequence.

a. -2, 6, 14, 22, 30, 38, 46
+8 +8 +8

b. 3, 0, -3, -6, -9, -12, -15
-3 -3 -3

c. You try: 40, 44, 48, 52, 56, 60, 64
+4 +4 +4

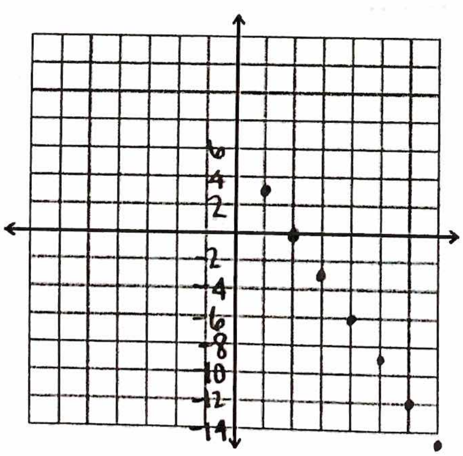
d. You try: 0.5, 1, 1.5, 2, 2.5, 3, 3.5
+0.5 +0.5 +0.5

I can graph arithmetic sequences

Example 2: Graph the arithmetic sequences from parts b and d above. What do you notice?

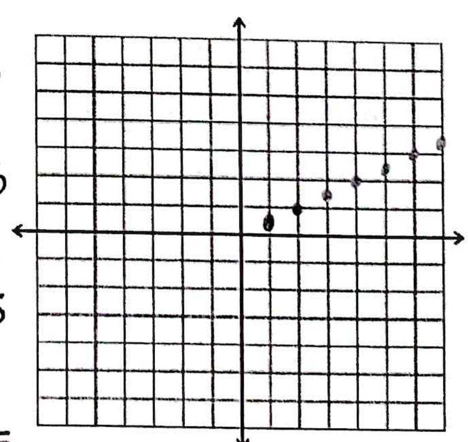
b.

a_1	3
a_2	0
a_3	-3
a_4	-6
a_5	-9
a_6	-12
a_7	-15



d.

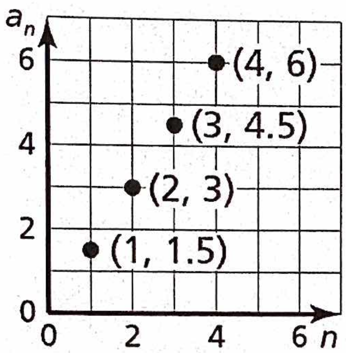
a_1	0.5
a_2	1
a_3	1.5
a_4	2
a_5	2.5
a_6	3
a_7	3.5



4.6 Arithmetic Sequences

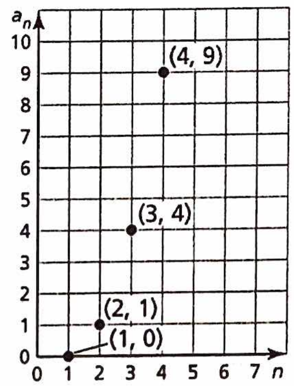
Example 3: Does the graph represent an arithmetic sequence? Explain.

a.



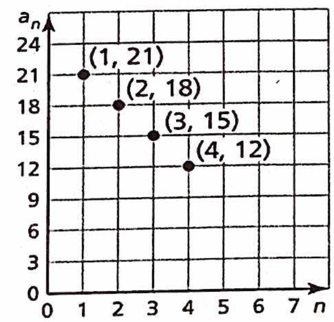
Yes - add 1.5

b.



No - no common difference

c. You try:



Yes - subtract 3

I Can Write Arithmetic Sequences as Functions

Since consecutive terms in an arithmetic sequence have a common different, the sequence has a constant rate of change.

Position, n	Term, a_n	Written using a_1 and d	Numbers
1	first term, a_1	a_1	4
2	second term, a_2	a_1+d	7
3	third term, a_3	a_1+2d	10
4	fourth term, a_4	a_1+3d	13
⋮	⋮	⋮	⋮
n	n^{th} term, a_n	$a_1+(n-1)d$	$a_1+(n-1)d$

From this, we have the equation for an arithmetic sequence: $a_n=a_1+(n-1)d$

Example 4: Write an equation for the nth term of the arithmetic sequence 7, 3, -1, -5, ... Then find a_{30}.

-4 -4 -4

$a_1=7$

$d=-4$

$a_n=7+(n-1)(-4)$

$a_n=7-4n+4$

$\boxed{a_n=-4n+11}$

$a_{30}=-4(30)+11$

$\boxed{a_{30}=-109}$

You try: Write an equation for the nth term of the arithmetic sequence 1, 0, -1, -2, ... Then find a_{51}.

-1 -1 -1

$a_1=1$

$d=-1$

$a_n=1+(n-1)(-1)$

$a_n=1-n+1$

$\boxed{a_n=-n+2}$

$a_{51}=-51+2$

$\boxed{a_{51}=-49}$

The equation for an arithmetic sequence can also be written using function notation by replacing a_n with $f(n)$.

4.6 Arithmetic Sequences

Example 5: Juan saves $12 the first week of the year. He will increase the amount he saves each week by $4.

a_1

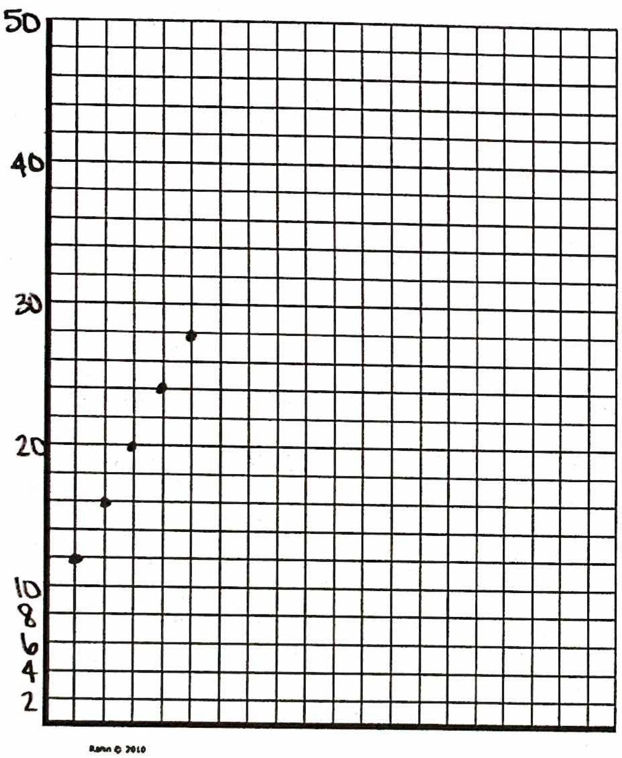
d

a. Write a function that represents the arithmetic sequence. $a_n = 12 + (n-1)4$

b. Graph the function.

$a_n = 12 + 4n - 4$

$\boxed{a_n = 4n + 8}$



a_1	12
a_2	16
a_3	20
a_4	24
a_5	28

c. Juan's goal is to save $68 in one week. In which week will he save that amount? 15

$68 = 4n + \cancel{8}$

$-8 \qquad \cancel{-8}$

$\frac{60}{4} = \frac{4n}{4}$

$n = 15$

6.6 Geometric Sequences

Learning Target

- I can identify patterns of arithmetic sequences and geometric sequences.

Success Criteria

- I can identify geometric sequences.
- I can extend and graph geometric sequences
- I can write geometric sequences as functions.

I can identify geometric sequences

Geometric Sequence: a sequence in which the ratio between each pair of consecutive terms is the same. The ratio is known as the common ratio. It is the number that you multiply by to get the next term.

Example 1: Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

a) 8, 24, 72, 216, ...

x3 x3 x3

geometric - multiplies by 3

b) 8, 3, -2, -7, ...

-5 -5 -5

arithmetic - subtracts 5

c) 1024, 128, 16, 2, ...

$\times\frac{1}{8}$ $\times\frac{1}{8}$ $\times\frac{1}{8}$

geometric - multiplies by $\frac{1}{8}$

I can extend and graph geometric sequences

Example 2: (You try) Write the next three terms of each geometric sequence.

a) 7, -14, 28, -56, 112, -224, 448

x-2 x-2 x-2

b) 1, 3, 9, 27, 81, 243, 729

x3 x3 x3

c) 16, 12, 8, 4, 0, -4, -8 *arithmetic

-4

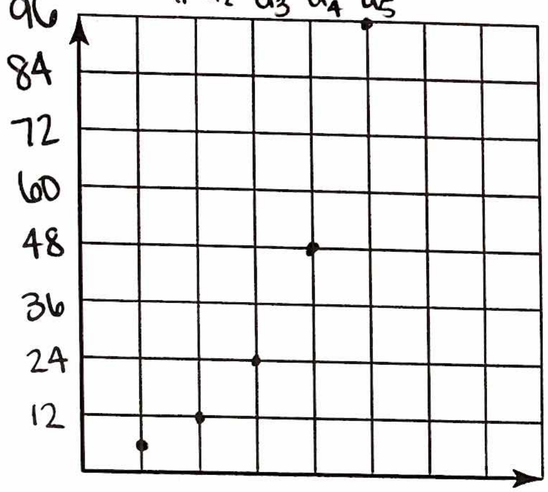
6.6 Geometric Sequences

I can extend and graph geometric sequences (cont.)

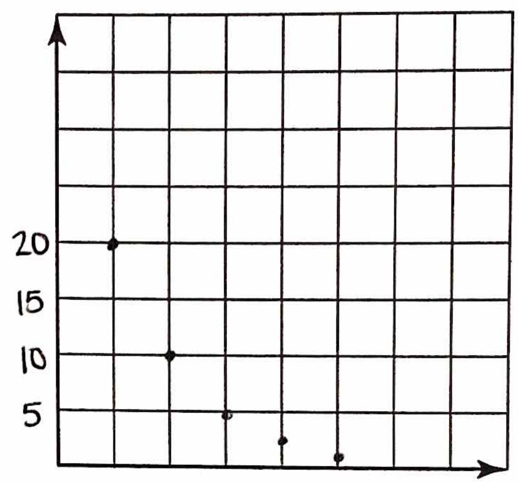
Example 3: Graph each geometric sequence. What do you notice?

exponential function

a) 6, 12, 24, 48, 96, …
a_1 a_2 a_3 a_4 a_5



b) 20, 10, 5, 5/2, 5/4, …



I can write geometric sequences as functions

Position, n	Term, a_n	Written using a_1 and r	Numbers (Let $a_1 = 1$, $r = 4$)
1	first term, a_1	a_1	1
2	second term, a_2	$a_1 \cdot r$	4
3	third term, a_3	$a_1 \cdot r \cdot r = a_1 r^2$	16
4	fourth term, a_4	$a_1 \cdot r \cdot r \cdot r = a_1 r^3$	64
⋮			
n	n^{th} term, a_n	$a_1 r^{n-1}$	$1(4)^{n-1}$

From this, we have the equation for the *n*th term of a geometric sequence: $a_n = a_1 (r)^{n-1}$

Example 4: Write an equation for the *n*th term of the geometric sequence 3, 12, 48, 192, …
Then find a_{10}.

x4 x4

$a_1 = 3$

$r = 4$

$\boxed{a_n = 3(4)^{n-1}}$

$a_{10} = 3(4)^{10-1}$

$\boxed{a_{10} = 786432}$

You try: Write an equation for the *n*th term of the geometric sequence 13, 26, 52, 104, … Then find a_{10}.

$a_1 = 13$

$r = 2$

$\boxed{a_n = 13(2)^{n-1}}$

$a_{10} = 13(2)^{10-1}$

$\boxed{a_{10} = 6656}$

6.6 Geometric Sequences

I can write geometric sequences as functions (cont.)

Note: You can write a geometric sequence in function notation by replacing a_n with $f(n)$.

Example 5:

An archery competition begins with $\underset{}{\overset{a_1}{256}}$ competitors. After the first round, $\overset{r}{\text{one-fourth}}$ of the competing group remains. After the second round, one-fourth of the now smaller competing group remains. The last round is when there are fewer than five members in the competing group.

a. Which round is the last round?

a_1 256
a_2 64
a_3 16
a_4 4

$\boxed{\text{4th round}}$

$\boxed{a_n = 256\left(\frac{1}{4}\right)^{n-1}}$

b. How many competitors are in the last round?

4

You try: A digital city map displays an area of $\overset{a_1}{544}$ square units. After you zoom in once, the area is 272 square units. After you zoom in a second time, the area is 136 square units. What is the area after you zoom in five times?

$a_1 =$ 544
a_2 272
a_3 136
a_4 68
a_5 34
a_6 17

$\boxed{a_n = 544\left(\frac{1}{2}\right)^{n-1}}$

$\boxed{\text{17 sq mi}}$