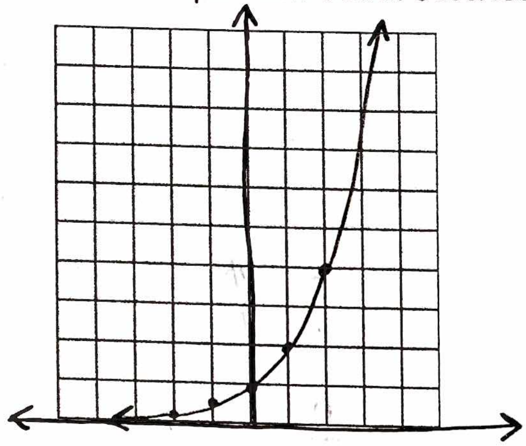
6.3 Exponential Functions

Class Example

Graph $f(x) = (2)^x$. Answer the questions below. Describe the domain and range of f.

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4



D: $\mathbb{R}$

R: $y>0$

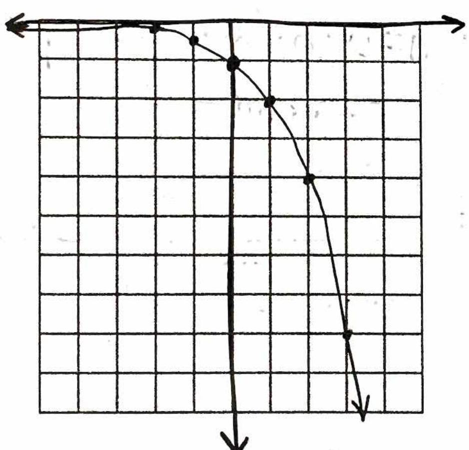
What pattern exists in the table? multiply by 2

What is the y-intercept? 1

Group Example

Graph $f(x) = -(2)^x$. Answer the questions below. Describe the domain and range of f.

x	y
-2	$-\frac{1}{4}$
-1	$-\frac{1}{2}$
0	-1
1	-2
2	-4



D: $\mathbb{R}$

R: $y<0$

What pattern exists in the table? multiply by 2

What is the y-intercept? -1

What is similar to the class example? Table: multiply by 2

Graph: L shape

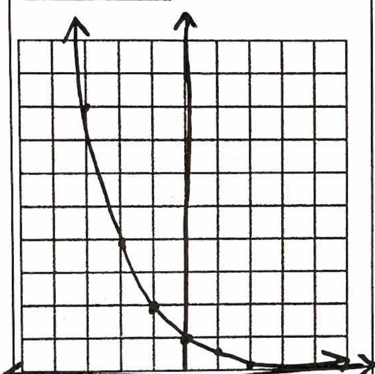
What is different from the class example? Table: all negative

Graph: reflection → top going up
bottom going down

6.3 Exponential Functions

$f(x) = (\frac{1}{2})^x$

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

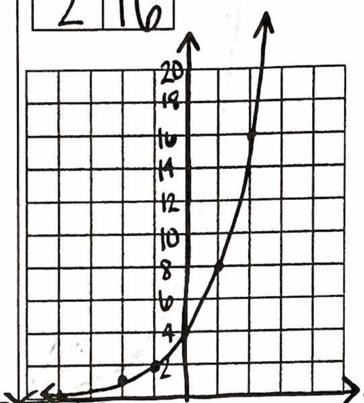


D: $\mathbb{R}$

R: $y>0$

$f(x) = 4(2)^x$

x	y
-2	1
-1	2
0	4
1	8
2	16

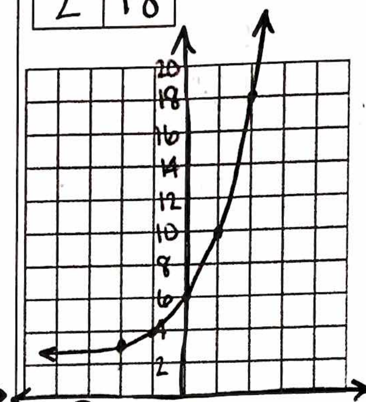


D: $\mathbb{R}$

R: $y>0$

$f(x) = 4(2)^x+2$

x	y
-2	3
-1	4
0	6
1	10
2	18

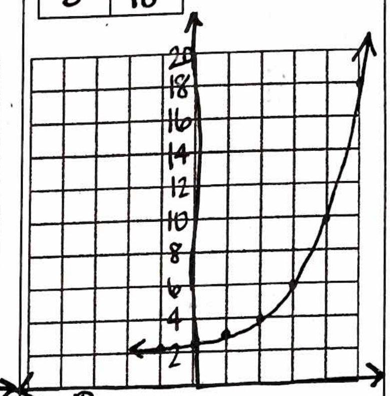


D: $\mathbb{R}$

R: $y>2$

$f(x) = 4(2)^{x-3}+2$ -1   2.25   0   2.5

x	y
1	3
2	4
3	6
4	10
5	18



D: $\mathbb{R}$

R: $y>2$

Summary of Graphing Exponential Functions

$y=a(b)^x$

a → y-intercept

b → what table is multiplied by

b>1 → increases   b<1 → decreases

a negative → reflection

$y=a(b)^x+c$

+c → shifts up

−c → shifts down

$y=a(b)^{x+d}$

+d → shifts left

−d → shifts right

Domain: $\mathbb{R}$ Range: $y>d$

Criterion B: Investigating Pattern Rubric

Assessing ii) **Describe** patterns as general rules consistent with findings

Achievement Level	
0	
1-2 The student is able to:	**State** predictions consistent with patterns about graphing exponential functions and its properties.
3-4	**Suggest** general rules consistent with findings about graphing exponential functions and its properties.
5-6	**Describe** patterns between the tables and graphs of exponential functions as general rules consistent with findings to create a summary of the relationships that is mostly correct.
7-8	**Describe** patterns between the tables and graphs of exponential functions as general rules consistent with correct findings to create a summary of the relationships.

6.3 Exponential Functions

Exponential Functions

$$y = ab^x$$

Transformations of Exponential Functions

$$y = ab^{x-h} + k$$

a	h	k
y-intercept	$+h \rightarrow$ left $-h \rightarrow$ right	$+k \rightarrow$ up $-k \rightarrow$ down

Domain: $\mathbb{R}$

Range: $y > k$

Identifying functions

Does each table represent a *linear* or an *exponential* function? Explain.

1)

+1 +1 +1

x	0	1	2	3
y	8	4	2	1

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

exponential - rate is $\frac{1}{2}$

2)

+4 +4 +4

x	−4	0	4	8
y	1	0	−1	−2

−1 −1 −1

linear - constant rate of change

Evaluating Exponential Functions

Evaluate each function for the given value of x. PEMDAS → exponents before multiplication

1) $y = -2(5)^x$; $x = 3$

$y = -2(5)^3$

$y = -2(125)$ $\boxed{y = -250}$

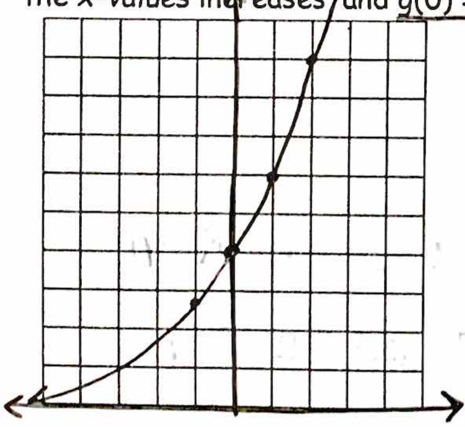
2) $y = 3(0.5)^x$; $x = -2$

$y = 3(0.5)^{-2}$

$y = 3(4)$ $\boxed{y = 12}$

Writing Exponential Functions

An exponential function *g* models a relationship where the *y*-values are multiplied by 1.5 for every 1 unit the *x-values* increases and $g(0) = 4$. Write an equation that represents the function. Graph the function



x	y
-1	$2.\overline{6}$
0	4
1	6
2	9
3	13.5

$a = 4$

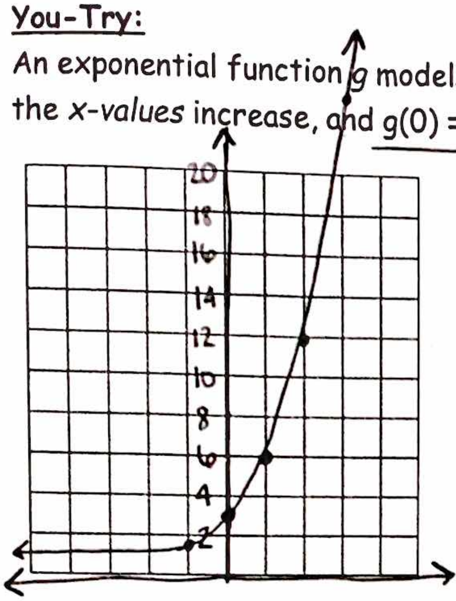
$b = 1.5$

$y = 4(1.5)^x$

6.3 Exponential Functions

You-Try:

An exponential function g models a relationship where the y-values are multiplied by 2 for every 1 unit the x-values increase, and $g(0) = 3$. Write an equation that represents the function. Graph the function



x	y
-1	1.5
0	3
1	6
2	12
3	24

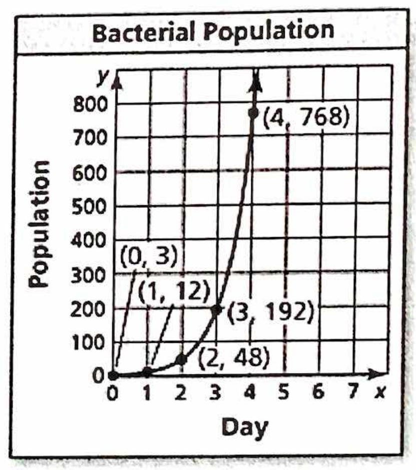
$a=3$

$b=2$

$y=3(2)^x$

Example 7: Modeling with Mathematics

The graph represents a bacterial population y after x days.



a) Write an exponential function that represents the population.

multiplies by 4 → $b=4$

$a=3$

$\boxed{y=3(4)^x}$

b) Find the population after 12 hours and after 5 days.

12 hours → $\boxed{x=0.5}$

$y=3(4)^{0.5}$

$\boxed{y=6}$

5days → $\boxed{x=5}$

$y=3(4)^5$

$\boxed{y=3072}$

You try:

A bacterial population y after x days can be represented by an exponential function whose graph passes through (0, 100) and (1, 200).

a) Write a function that represents the population.

$a=100$

$b=2$

$\boxed{y=100(2)^x}$

b) Find the population after 6 days.

$y=100(2)^6$ $\boxed{y=6400}$

c) Does this bacterial population grow faster than the bacterial population in the previous example? Explain.

No- the base is 2 in this example so it doubles where in the previous example the base is 4 so its quadrupled.

6.4 Exponential Growth and Decay

Learning Target:

- I can graph exponential functions and solve exponential equations.

Success Criteria:

- I can use and identify exponential growth and decay functions.
- I can interpret and rewrite exponential growth and decay functions.
- I can solve real-life problems involving exponential growth and decay.

Exponential Growth:

rate (as a decimal)

initial amount $y = a(1 + r)^t$

time

Exponential Decay:

$$y = a(1 - r)^t$$

Example 1: Identifying Exponential Growth and Decay

Determine whether each table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

1)

x	y
0	24
1	12
2	6
3	3

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

$\boxed{\text{exponential decay}}$

2)

x	0	1	2	3
y	4	40	400	4000

x10 x10 x10

exponential growth

3)

x	0	1	2	3
y	64	16	4	1

$\times\frac{1}{4}$ $\times\frac{1}{4}$ $\times\frac{1}{4}$

$\boxed{\text{exponential decay}}$

4)

x	1	3	5	7
y	4	11	18	25

+7 +7 +7

$\boxed{\text{neither - linear}}$

Example 2: Writing Exponential Functions

The inaugural attendance of an annual music festival is 80,000. The attendance y increases by 6% each year. .06

a) Write an exponential function that represents the attendance after t years.

$y = 80{,}000(1 + .06)^x$ $\boxed{y = 80000(1.06)^t}$

b) How many people will attend the festival in the 5th year? Round your answer the nearest thousand.

$y = 80000(1.06)^5$ $\boxed{y = 107058 \text{ people}}$

You purchase a car for $25,000. The value of a car depreciates by 15% each year. 0.15

a) Write an exponential function that represents the value after x years.

$y = 25000(1 - .15)^x$ $y = 25000(0.85)^x$

b) How much will the car be worth after 4 years?

$y = 25000(.85)^4$ $\boxed{y = 13050.16}$

6.5 Solving Exponential Equations

Learning Target

- I can graph exponential functions and solve exponential equations.

Success Criteria

- I can solve exponential equations with the same base
- I can solve exponential equations with unlike bases
- I can solve exponential equations by graphing

I can solve exponential equations with the same base

Let's learn this process through an example:

1) $2^{2x} = 2^6$ *Note the variable is in the exponent! There is only one way this can be true....

....the exponents must be equal!

$\frac{2x}{2} = \frac{6}{2}$

$\boxed{x=3}$

2) $5^{2x} = 5^{x+1}$

$2x = \cancel{x}+1$

$-x \quad \cancel{-x}$

$\boxed{x=1}$

3) $7^{3x+5} = 7^{x+1}$

$3x+5 = \cancel{x}+1$

$-x \quad \cancel{-x}$

$2x+\cancel{5} = 1$

$\cancel{-5} \quad -5$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{-4}{2}$

$\boxed{x=-2}$

I can solve exponential equations with unlike bases

These take a bit more work, we need to make the bases the same before we set exponents equal

1) $4^x = 256$

$4^x = 4^4$

$\boxed{x=4}$

2) $9^{2x} = 3^{x-6}$

$3^{2(2x)} = 3^{x-6}$

$2(2x) = x-6$

$4x = \cancel{x}-6$

$-x \quad \cancel{-x}$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-6}{3}$

$\boxed{x=-2}$

3) $4^{3x} = 8^{x+1}$

$2^{2(3x)} = 2^{3(x+1)}$

$2(3x) = 3(x+1)$

$6x = \cancel{3x}+3$

$-3x \quad \cancel{-3x}$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{3}{3}$

$\boxed{x=1}$

You try:

1) $2^x = 64$

$2^x = 2^6$

$\boxed{x=6}$

2) $7^{x-5} = 49^x$

$7^{x-5} = 7^{2x}$

$\cancel{x}-5 = 2x$

$\cancel{-x} \quad -x$

$\boxed{-5 = x}$

3) $64^{2x+4} = 16^{5x}$

$4^{3(2x+4)} = 4^{2(5x)}$

$3(2x+4) = 2(5x)$

$\cancel{6x}+12 = 10x$

$\cancel{-6x} \quad -6x$

$\frac{12}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$\boxed{x=3}$

6.5 Solving Exponential Equations

Let's add another challenge…what happens when the base is a fraction!

1) $\left(\frac{1}{3}\right)^{x-1} = 27$

$3^{-1(x-1)} = 3^3$

$-1(x-1) = 3$

$-x+\cancel{1} = 3$

$\cancel{-1} \quad -1$

$\frac{-x}{-1} = \frac{2}{-1}$

$\boxed{x=-2}$

2) $\frac{1}{128} = 2^{5x+3}$

$2^{-7} = 2^{5x+3}$

$-7 = 5x+\cancel{3}$

$-3 \quad \cancel{-3}$

$\frac{-10}{5} = \frac{\cancel{5}x}{\cancel{5}}$

$\boxed{x=-2}$

You try:

1) $\left(\frac{1}{5}\right)^{x} = 125$

$5^{-1(x)} = 5^3$

$\frac{\cancel{-1}x}{\cancel{-1}} = \frac{3}{-1}$

$\boxed{x=-3}$

2) $36^{-3x+3} = \left(\frac{1}{6}\right)^{x-1}$

$6^{2(-3x+3)} = 6^{-1(x-1)}$

$2(-3x+3) = -1(x-1)$

$\cancel{-6x}+6 = -1x+1$

$\cancel{+6x} \quad +6x$

$6 = 5x+\cancel{1}$

$-1 \quad \cancel{-1}$

$\frac{5}{5} = \frac{5x}{5}$

$\boxed{x=1}$

<u>*I can solve exponential equations by graphing*</u>

Make sure you have your graphing calculator!

1) $2^x = 1.8$

$\boxed{x=0.85}$

2) $4^{x-3} = x+2$

$\boxed{x=-1.999}$

$\boxed{x=4.33}$

3) $\left(\frac{1}{4}\right)^{x} = -2x-3$

$\boxed{\text{no solution}}$

You try:

1) $6^{x+2} = 12$

$\boxed{x=-0.61}$

2) $\left(\frac{1}{2}\right)^{7x+1} = -9$

$\boxed{\text{no Solution}}$

3) $2^{x+6} = 2x+5$

$\boxed{\text{no solution}}$