9.1 Properties of Radicals

Learning Targets:

- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can use properties of radicals to simplify expressions
- I can simplify expressions by rationalizing the denominator
- I can perform operations with radicals

Opener Problem

a) Square Roots and Addition

Is $\sqrt{36}+\sqrt{64}$ the same as $\sqrt{36+64}$? In general, is $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$?

$6+8$ $\sqrt{100}$

14 10 No

b) Square Roots and Multiplication

Is $\sqrt{36}\cdot\sqrt{64}$ the same as $\sqrt{36\cdot 64}$? In general, is $\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$?

$6\cdot 8$ $\sqrt{2304}$

48 48 Yes

c) Square Roots and Subtraction

Is $\sqrt{64}-\sqrt{36}$ the same as $\sqrt{64-36}$? In general, is $\sqrt{a}-\sqrt{b}=\sqrt{a-b}$?

$8-6$ $\sqrt{28}$ No

2 $5.29\ldots$

d) Square Roots and Division

Is $\frac{\sqrt{100}}{\sqrt{4}}$ the same as $\sqrt{\frac{100}{4}}$? In general, is $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$? Yes

$\frac{10}{2}=5$ $\sqrt{25}=5$

Properties of Radicals

Radical Expression - An expression that contains a radical ($\sqrt{\ }$) radicand

Simplest Form -
- No radicands have perfect n^{th} powers as factors other than 1
- No radicands contain fractions
- No radicals in denominator (No rats in the basement)

9.1 Properties of Radicals

I can use properties of radicals to simplify expressions:

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

☆ Look for highest perfect square that divides into radicand *

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ **Algebra** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Notes - these properties are also true for cube roots

Examples:

a) $\sqrt{24}$ $\sqrt{4 \cdot 6}$ $\sqrt{4}\sqrt{6}$ $\boxed{2\sqrt{6}}$	b) $\sqrt{162g^6}$ $\sqrt{81 \cdot 2 \cdot g^2 \cdot g^2 \cdot g^2}$ $\sqrt{81}\sqrt{2}\sqrt{g^2}\sqrt{g^2}\sqrt{g^2}$ $9\sqrt{2}\ g \cdot g \cdot g$ $\boxed{9g^3\sqrt{2}}$	c) $-\sqrt{80}$ $-\sqrt{16 \cdot 5}$ $-\sqrt{16} \cdot \sqrt{5}$ $\boxed{-4\sqrt{5}}$	d) $\sqrt{49x^3}$ $\sqrt{49 \cdot x^2 \cdot x}$ $\sqrt{49} \cdot \sqrt{x^2} \cdot \sqrt{x}$ $\boxed{7x\sqrt{x}}$
e) $-\sqrt{\frac{6}{49}}$ $\frac{-\sqrt{6}}{\sqrt{49}}$ $\boxed{\frac{-\sqrt{6}}{7}}$	f) $-\sqrt{\frac{196}{r^4}}$ $\frac{-\sqrt{196}}{\sqrt{r^2 \cdot r^2}}$ $\boxed{\frac{-14}{r^2}}$	g) $-\sqrt{\frac{17}{100}}$ $\frac{-\sqrt{17}}{\sqrt{100}}$ $\boxed{\frac{-\sqrt{17}}{10}}$	h) $\sqrt{\frac{4x^2}{64}}$ $\frac{\sqrt{4x^2}}{\sqrt{64}}$ $\frac{2x}{8} = \boxed{\frac{x}{4}}$

9.1 Properties of Radicals

You try:

a) $-\sqrt{48}$ $-\sqrt{16\cdot 3}$ $-\sqrt{16}\cdot\sqrt{3}$ $\boxed{-4\sqrt{3}}$	b) $-\sqrt{512h^7}$ $-\sqrt{256\cdot 2\cdot h^6\cdot h}$ $-\sqrt{256}\cdot\sqrt{2}\cdot\sqrt{h^6}\cdot\sqrt{h}$ $\boxed{-16h^3\sqrt{2h}}$	c) $\sqrt{147}$ $\sqrt{49\cdot 3}$ $\sqrt{49}\cdot\sqrt{3}$ $\boxed{7\sqrt{3}}$
d) $\sqrt{\frac{8}{100}}$ OR $\frac{\sqrt{2}}{\sqrt{25}}$ $\frac{\sqrt{4\cdot 2}}{\sqrt{100}}$ $\frac{\sqrt{2}}{5}$ $\frac{2\sqrt{2}}{10}$ $\boxed{\frac{\sqrt{2}}{5}}$	e) $\sqrt{\frac{25}{64}}$ $\frac{\sqrt{25}}{\sqrt{64}}$ $\boxed{\frac{5}{8}}$	f) $\sqrt{\frac{49x^3}{64y^2}}$ $\frac{\sqrt{49\cdot x^2\cdot x}}{\sqrt{64\cdot y^2}}$ $\boxed{\frac{7x\sqrt{x}}{8y}}$

I can simplify expressions by rationalizing the denominator: No rats in the basement

Rationalizing the Denominator - when radical is in the denominator, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator

Examples:

a) $\frac{1}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$ $\boxed{\frac{\sqrt{7}}{7}}$	b) $\frac{\sqrt{8}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{24}}{3}$ $\frac{\sqrt{4\cdot 6}}{3}$ $\boxed{\frac{2\sqrt{6}}{3}}$	c) $\frac{6}{\sqrt{3x}}\cdot\frac{\sqrt{3x}}{\sqrt{3x}}$ $\frac{6\sqrt{3x}}{3x}$ $\boxed{\frac{2\sqrt{3x}}{x}}$	d) $\sqrt{\frac{2y^2}{3}} = \frac{\sqrt{2y^2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{6y}}{3}$ $\boxed{\frac{y\sqrt{6}}{3}}$

9.1 Properties of Radicals

You try:

a) $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $\boxed{\frac{\sqrt{5}}{5}}$	b) $\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\boxed{\frac{\sqrt{14}}{2}}$	c) $\frac{5}{\sqrt{6x}} \cdot \frac{\sqrt{6x}}{\sqrt{6x}}$ $\boxed{\frac{5\sqrt{6x}}{6x}}$	d) $\sqrt{\frac{8r^2}{5}} = \frac{\sqrt{8r^2}}{\sqrt{5}}$ $\frac{\sqrt{4 \cdot 2 \cdot r^2}}{\sqrt{5}}$ $\frac{2r\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $\boxed{\frac{2r\sqrt{10}}{5}}$

I can perform operations with radicals

"like" radicals - Radicals with same index and radicand

Example:

a) $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$ $\boxed{13\sqrt{2} - \sqrt{6}}$	b) $3\sqrt{8} + 3\sqrt{2}$ $3\sqrt{4 \cdot 2} + 3\sqrt{2}$ $3 \cdot 2\sqrt{2} + 3\sqrt{2}$ $6\sqrt{2} + 3\sqrt{2}$ $\boxed{9\sqrt{2}}$	c) $2\sqrt{18} - 2\sqrt{20} - 2\sqrt{5}$ $2\sqrt{9 \cdot 2} - 2\sqrt{4 \cdot 5} - 2\sqrt{5}$ $2 \cdot 3\sqrt{2} - 2 \cdot 2\sqrt{5} - 2\sqrt{5}$ $6\sqrt{2} - 4\sqrt{5} - 2\sqrt{5}$ $\boxed{6\sqrt{2} - 6\sqrt{5}}$

9.1 Properties of Radicals

You try:

a) $4\sqrt{3} - 5\sqrt{5} + 12\sqrt{3}$ $\boxed{16\sqrt{3} - 5\sqrt{5}}$	b) $3\sqrt{12} + 3\sqrt{18} + 2\sqrt{27}$ $3\sqrt{4\cdot 3} + 3\sqrt{9\cdot 2} + 2\sqrt{9\cdot 3}$ $3\cdot 2\sqrt{3} + 3\cdot 3\sqrt{2} + 2\cdot 3\sqrt{3}$ $6\sqrt{3} + 9\sqrt{2} + 6\sqrt{3}$ $\boxed{12\sqrt{3} + 9\sqrt{2}}$

Multiplying Radicals

Examples:

You try:

$2\sqrt{5}(\sqrt{6} + 2)$ $\boxed{2\sqrt{30} + 4\sqrt{5}}$	$\sqrt{3}(8\sqrt{2} + 7\sqrt{32})$ $8\sqrt{6} + 7\sqrt{96}$ $8\sqrt{6} + 7\sqrt{16\cdot 6}$ $8\sqrt{6} + 7\cdot 4\sqrt{6}$ $8\sqrt{6} + 28\sqrt{6}$ $\boxed{36\sqrt{6}}$

__Closure__: **What I learned today was....**

9.2 Solving Quadratic Equations by Graphing

Learning Targets:

- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can solve quadratic equations by graphing
- I can use graphs to find and approximate the zeros of functions
- I can solve real-life problems using graphs of quadratic functions

I can solve quadratic equations by graphing

Step 1 Write the equation in standard form, $ax^2 + bx + c = 0$.

Solutions = roots = x-ints.

Step 2 Graph the related function $y = ax^2 + bx + c$.

Step 3 Find the x-intercepts, if any.

The solutions, or *roots*, of $ax^2 + bx + c = 0$ are the x-intercepts of the graph.

Number of Solutions of a Quadratic Equation

A quadratic equation has:

- two real solutions when the graph of its related function has 2 x-intercepts.
- one real solution when the graph of its related function has 1 x-intercept.
- no real solutions when the graph of its related function has 0 x-intercepts.

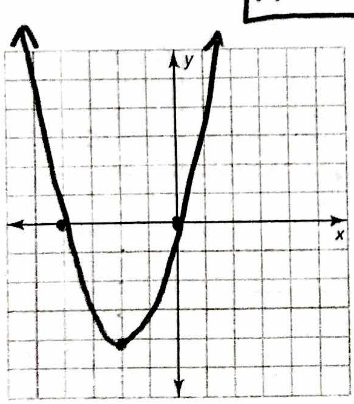
Example: Solve the equation by graphing (Note - use ANY method for graphing - table, vertex/y-int/symmetric point, etc)

a) $x^2 + 4x = 0$

$x = \frac{-4}{2} = -2$

$(-2, -4)$

$\boxed{x = -4, 0}$



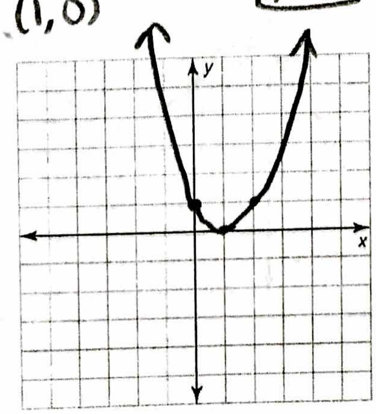
b) $-x^2 = -2x + 1$

$0 = x^2 - 2x + 1$

$x = \frac{2}{2} = 1$

$(1, 0)$

$\boxed{x = 1}$

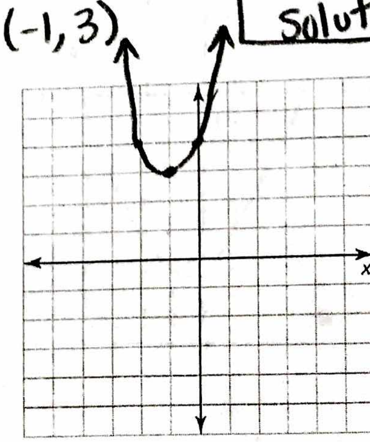


c) $x^2 + 2x + 4 = 0$

$x = \frac{-2}{2} = -1$

$(-1, 3)$

$\boxed{\text{no real Solutions}}$



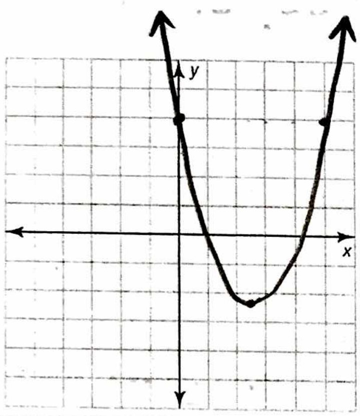
9.2 Solving Quadratic Equations by Graphing

d) $x^2 - 5x + 4 = 0$

$x = \frac{5}{2(1)} = 2.5$

$(2.5, -2.25)$

$\boxed{x = 1, 4}$

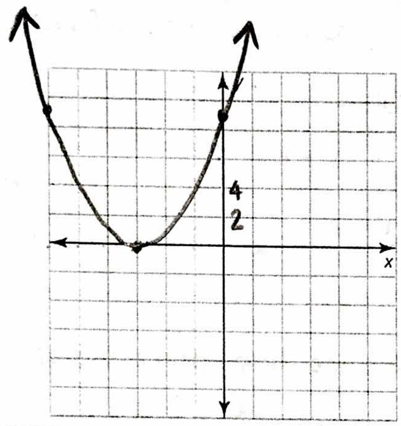


e) $x^2 + 6x + 9 = 0$

$x = \frac{-6}{2} = -3$

$(-3, 0)$

$\boxed{x = -3}$



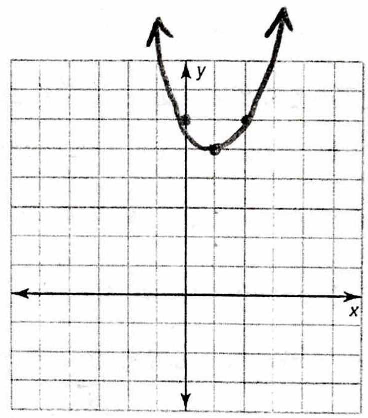
f) $x^2 = 2x - 6$

$x^2 - 2x + 6 = 0$

$x = \frac{2}{2} = 1$

$(1, 5)$

$\boxed{\text{no sol.}}$



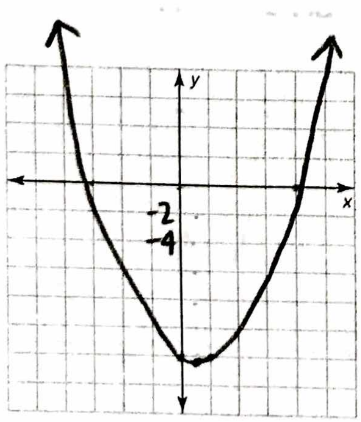
You try:

a) $x^2 - x - 12 = 0$

$x = \frac{1}{2}$

$(\frac{1}{2}, -12.25)$

$\boxed{x = -3, 4}$

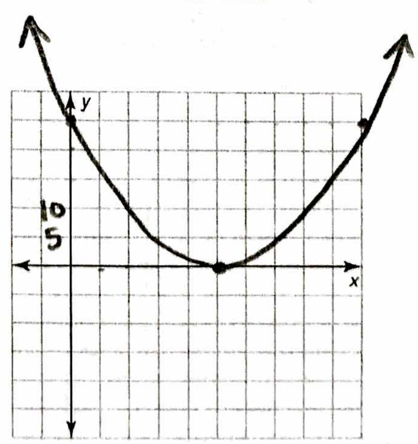


b) $x^2 - 10x + 25 = 0$

$x = \frac{10}{2} = 5$

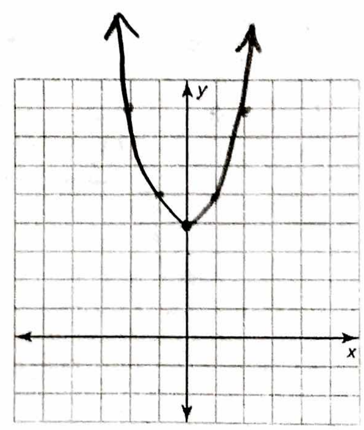
$(5, 0)$

$\boxed{x = 5}$



c) $x^2 + 4 = 0$

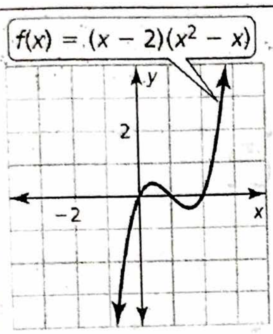
$\boxed{\text{no sol.}}$



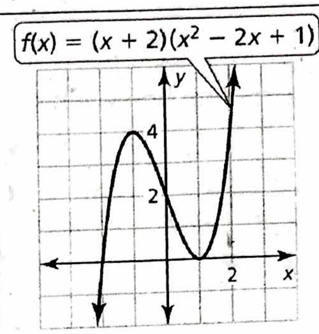
9.2 Solving Quadratic Equations by Graphing

I can use graphs to find and approximate the zeros of functions

Example: Using the graph, find the zeros.

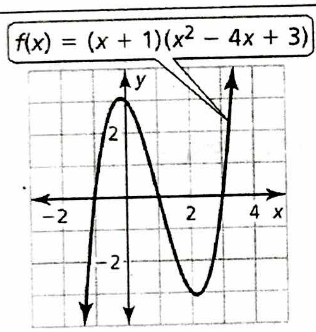


$X = 0, 1, 2$

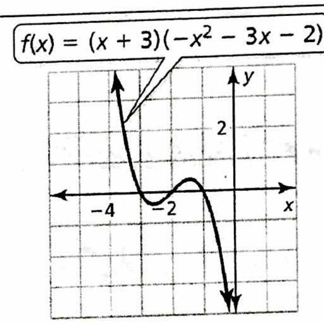


$X = -2, 1$

You try:



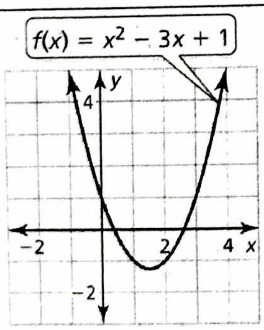
$X = -1, 1, 3$



$X = -3, -2, -1$

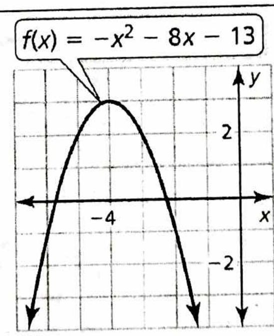
Approximate the zeros to the nearest tenth:

Example:



$X \approx 0.5, 2.5$

You try:



$X \approx -5.7, -2.2$

9.2 Solving Quadratic Equations by Graphing

I can solve real-life problemsm using graphs of quadratic functions

A soccer player kicks a soccer ball 2 feet above the ground with an initial vertical velocity of 60 feet per second. The function $h = -16t^2 + 60t + 2$ represents the height h (in feet) of the soccer ball after t seconds.

a) Find the height of the soccer ball each second after it is kicked.

0	1	2	3	4
2	46	58	38	-14

b) Use the results of part (a) to estimate when the height of the soccer ball is 40 feet.

0.8 sec, 2.9 sec.

c) Using a graph, after how many seconds is the soccer ball 40 feet above the groung?

$x \approx 0.807$ sec, 2.94 sec

Closure: What I learned today was...

9.3 Solving Quadratic Equations Using Square Roots

Learning Targets:

- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can solve quadratic equations using square roots
- I can approximate the solutions of quadratic equations

I can solve quadratic equations using square roots

Solutions of $x^2 = d$

- When $d > 0, x^2 = d$ has two real solutions, $x = \pm\sqrt{d}$.
- When $d = 0, x^2 = d$ has one real solution, $x = 0$.
- When $d < 0, x^2 = d$ has no real solutions.

Example: Solve the equation using square roots (NO DECIMAL ANSWERS!!)

a) $x^2 + \cancel{49} = 0$ $-\cancel{49}\quad -49$ $x^2 = -49$ $x = \sqrt{-49}$ $\boxed{\text{no real Sol.}}$	b) $x^2 + \cancel{6} = 6$ $-\cancel{6}\quad -6$ $x^2 = 0$ $x = \sqrt{0}$ $\boxed{x = 0}$	c) $2x^2 - 72 = 0$ $\frac{2x^2}{2} = \frac{72}{2}$ $x^2 = 36$ $x = \sqrt{36}$ $\boxed{x = \pm 6}$
d) $8x^2 - \cancel{49} = 151$ $+\cancel{49}\quad +49$ $8x^2 = 200$ $x^2 = 25$ $x = \sqrt{25}$ $\boxed{x = \pm 5}$	e) $81x^2 - \cancel{49} = -24$ $+\cancel{49}\quad +49$ $\frac{\cancel{81}x^2}{\cancel{81}} = \frac{25}{81}$ $x^2 = \frac{25}{81}$ $x = \sqrt{\frac{25}{81}}$ $\boxed{x = \pm\frac{5}{9}}$	f) $25x^2 + \cancel{9} = 0$ $-\cancel{9}\quad -9$ $\frac{\cancel{25}x^2}{\cancel{25}} = \frac{-9}{25}$ $x^2 = \frac{-9}{25}$ $\boxed{\text{no real sol.}}$
g) $\sqrt{(x - 4)^2} = \sqrt{0}$ $x - 4 = 0$ $\boxed{x = 4}$	h) $\sqrt{(4x - 3)^2} = \sqrt{64}$ $4x - 3 = \pm 8$ $4x - 3 = 8$ $\quad$ $4x - 3 = -8$ $4x = 11$ $\quad$ $4x = -5$ $\boxed{x = \frac{11}{4}}$ $\quad$ $\boxed{x = \frac{-5}{4}}$	i) $\cancel{16}(x - 3)^2 = 25$ $\cancel{16} \qquad 16$ $\sqrt{(x-3)^2} = \sqrt{\frac{25}{16}}$ $x - 3 = \pm\frac{5}{4}$ $x - 3 = \frac{5}{4}$ $\quad$ $x - 3 = -\frac{5}{4}$ $\boxed{x = \frac{17}{4}}$ $\quad$ $\boxed{x = \frac{7}{4}}$

9.3 Solving Quadratic Equations Using Square Roots

You try:

a) $x^2 - 25 = 0$ $x^2 = 25$ $x = \sqrt{25}$ $\boxed{x = \pm 5}$	b) $2x^2 + 84 = 0$ $2x^2 = -84$ $x^2 = -42$ $\boxed{\text{no real sol.}}$	c) $-x^2 - 12 = -12$ $-x^2 = 0$ $x^2 = 0$ $x = \sqrt{0}$ $\boxed{x = 0}$
d) $-3x^2 + 16 = -11$ $-3x^2 = -27$ $x^2 = 9$ $x = \sqrt{9}$ $\boxed{x = 3, -3}$	e) $16x^2 - 1 = 0$ $16x^2 = 1$ $x^2 = \frac{1}{16}$ $x = \sqrt{\frac{1}{16}}$ $\boxed{x = \pm \frac{1}{4}}$	f) $16 - 2x^2 = 16$ $-2x^2 = 0$ $x^2 = 0$ $x = \sqrt{0}$ $\boxed{x = 0}$
g) $(x + 2)^2 = 196$ $x + 2 = 14$ $\boxed{x = 12}$	h) $(2x + 7)^2 = 49$ $2x + 7 = 7$ $2x = 0$ $\boxed{x = 0}$	i) $81(3x + 1)^2 = 49$ $(3x+1)^2 = \frac{49}{81}$ $3x + 1 = \sqrt{\frac{49}{81}}$ $3x + 1 = \pm\frac{7}{9}$

$3x+1 = \frac{7}{9}$

$3x = \frac{-2}{9}$

$\boxed{x = \frac{-2}{27}}$

$3x+1 = -\frac{7}{9}$

$3x = \frac{-16}{9}$

$\boxed{x = \frac{-16}{27}}$

Real Life Problem: Example: A ball is dropped from a window at a height of 81 feet. The function $h = -16x^2 + 81$ represents the height (in feet) of the ball after x seconds. How long does it take for the ball to hit the ground?

$0 = -16x^2 + 81$

$\frac{-81}{-16} = \frac{\cancel{-16}x^2}{\cancel{-16}}$

$x^2 = \frac{81}{16}$

$x = \sqrt{\frac{81}{16}}$

$x = \pm \frac{9}{4}$

$\boxed{x = 2.25 \text{ sec}}$

9.3 Solving Quadratic Equations Using Square Roots

<u>I can approximate the solutions of quadratic equations</u>

Example: Solve the equation using square roots, round your solution to the nearest HUNDRETH.

a) $x^2 + 6 = 8$ $x^2 = 2$ $x = \sqrt{2}$ $\boxed{x \approx \pm 1.41}$	b) $3x^2 - 4 = 14$ $3x^2 = 18$ $x^2 = 6$ $x = \sqrt{6}$ $\boxed{x \approx \pm 2.45}$	c) $20 - 4x^2 = 18$ $-4x^2 = -2$ $x^2 = \frac{1}{2}$ $x = \sqrt{\frac{1}{2}}$ $\boxed{x \approx \pm 0.71}$

You try:

a) $x^2 - 12 = 3$ $x^2 = 15$ $x = \sqrt{15}$ $\boxed{x \approx \pm 3.87}$	b) $x^2 + 25 = 49$ $x^2 = 24$ $x = \sqrt{24}$ $\boxed{x \approx \pm 4.90}$	c) $6x^2 + 5 = 20$ $6x^2 = 15$ $x^2 = \frac{15}{6}$ $x = \sqrt{\frac{15}{6}}$ $\boxed{x \approx \pm 1.58}$

<u>Closure</u>: What I learned today was...