Key

10.3 Solving Radical Equations

Essential Questions:

How can you solve an equation that contains square roots?

Learning Goals:

- I can solve radical equations.
- I can identify extraneous solutions.
- I can solve real-life problems involving radical equations.

A radical equation is an equation that contains a radical expression with a variable in the radicand.

*If two expressions are equal, then their squares are also equal.

I can solve radical equations

Example 1: Solving Radical Equations

1) $\sqrt{x} = 6$

$(\sqrt{x})^2 = 6^2$

$\boxed{x = 36}$

2) $10 - \sqrt{z} = -5$

$-10 \quad -10$

$\frac{-\sqrt{z}}{-1} = \frac{-15}{-1}$

$(\sqrt{z})^2 = (15)^2$

$\boxed{z = 225}$

3) $3\sqrt{x+2} + \cancel{3} = 18$

$-\cancel{3} \quad -3$

$\frac{3\sqrt{x+2}}{3} = \frac{15}{3}$

$(\sqrt{x+2})^2 = 5^2$

$x + 2 = 25$

$\boxed{x = 23}$

You-Try:

1) $\sqrt{x} - 7 = 3$

$+7 \quad +7$

$\sqrt{x}^2 = 10^2$

$\boxed{x = 100}$

2) $1 - \sqrt{x} = -2$

$-1 \quad -1$

$-\sqrt{x} = -3$

$\sqrt{x} = 3$

$\boxed{x = 9}$

3) $15 = 6 + \sqrt{3w - 9}$

$-6 \quad -6$

$9^2 = \sqrt{3w-9}^2$

$81 = 3w - 9$

$90 = 3w$

$\boxed{30 = w}$

Example 2: Solving a Radical Equation with Radicals on Both Sides

1) $\sqrt{4x - 3}^2 = \sqrt{x + 9}^2$

$4x - 3 = x + 9$

$-x \quad -x$

$3x - 3 = 9$

$+3 \quad +3$

$3x = 12$

$\boxed{x = 4}$

2) $\sqrt{2x - 1}^2 = \sqrt{x + 4}^2$

$2x - 1 = x + 4$

$-x \quad -x$

$x - 1 = 4$

$+1 \quad +1$

$\boxed{x = 5}$

3) $\sqrt{3x + 1}^2 = \sqrt{4x - 7}^2$

$3x + 1 = 4x - 7$

$-3x \quad -3x$

$1 = x - 7$

$+7 \quad +7$

$\boxed{8 = x}$

10.3 Solving Radical Equations

You-Try:

1) $\sqrt{n}^2 = \sqrt{5n-1}^2$

$n = 5n - 1$
$-5n \quad -5n$
$\frac{-4n}{-4} = \frac{-1}{-4}$
$\boxed{n = \frac{1}{4}}$

2) $\sqrt{8h-7}^2 = \sqrt{6h+7}^2$

$8h - 7 = 6h + 7$
$-6h \quad -6h$
$2h - 7 = 7$
$+7 \quad +7$
$2h = 14$
$\boxed{h = 7}$

Example 3: Solving a Radical Equation Involving a Cube Root

1) $\sqrt[3]{2x-5}^3 = 3^3$

$2x - 5 = 27$
$+5 \quad +5$
$\frac{2x}{2} = \frac{32}{2}$
$\boxed{x = 16}$

2) $\sqrt[3]{y-4}^3 = 1^3$

$y - 4 = 1$
$+4 \quad +4$
$\boxed{y = 5}$

You-Try:

1) $\sqrt[3]{3c+7}^3 = 10^3$

$3c + 7 = 1000$
$-7 \quad -7$
$\frac{3c}{3} = \frac{993}{3}$
$\boxed{c = 331}$

2) $\sqrt[3]{y+6}^3 = \sqrt[3]{5y-2}^3$

$y + 6 = 5y - 2$
$-y \quad -y$
$6 = 4y - 2$
$+2 \quad +2$

$\frac{8}{4} = \frac{4y}{4}$
$\boxed{2 = y}$

I can identify extraneous solutions

Example 4: Identifying an Extraneous Solution

*extraneous solutions = solutions that doesn't satisfy the original equation when checked

1) $\sqrt{x}^2 = (-2)^2$

$x = 4$
$\sqrt{4} = -2$
$2 \neq -2$

$\sqrt{x}$ can never be negative → extraneous

2) $x^2 = \sqrt{x+6}^2$

$x^2 = x + 6$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$\boxed{x = 3 \quad x = -2}$

$3 = \sqrt{3+6}$ $-2 = \sqrt{-2+6}$
$3 = \sqrt{9}$ $-2 = \sqrt{4}$
$3 = 3$ ✓ $-2 \neq 2$
extraneous

3) $13 + \sqrt{5n} = 3$
$-13 \quad -13$

$\sqrt{5n} = -10$
$5n = 100$
$\boxed{n = 20}$

$13 + \sqrt{5(20)} = 3$
$13 + \sqrt{100} = 3$
$13 + 10 = 3$
$23 \neq 3$
extraneous

10.3 Solving Radical Equations

You-Try:

1) $\sqrt{4-3x}^2 = x^2$

$4-3x = x^2$

$0 = x^2+3x-4$

$(x+4)(x-1)=0$

$x=-4 \quad x=1$

$4-3(-4))=-4$

$\sqrt{4+12} = -4$

$\sqrt{16} = -4$

$4 \neq -4$

extraneous

2) $\sqrt{3m} + 10 = 1$

$-10 \quad -10$

$\sqrt{3m}^2 = -9^2$

$3m = 81$

$m = 27$

$\sqrt{3(27)}+10 = 1$

$\sqrt{81}+10=1$

$9+10=1$

$19 \neq 1$

extraneous

3) $(p+1)^2 = \sqrt{7p+15}^2$

$p^2+2p+1 = 7p+15$

$p^2-5p-14=0$

$(p-7)(p+2)=0$

$p=7 \quad p=-2$

extraneous

<u>I can solve real-life problems involving radical equations</u> Skip

<u>Example 5</u>: The time t (in seconds) it takes a trapeze artist to swing back and forth is represented by the function $t = 2\pi\sqrt{\frac{r}{32}}$, where r is the rope length (in feet). It takes the trapeze artist 6 seconds to swing back and forth. Is this rope $\frac{3}{2}$ as long as the rope used when it takes the trapeze artist 4 seconds to swing back and forth?

physics problems eliminated from chapter

<u>Closure:</u> What I learned today was……

10.4 Inverse of a Function

Essential Questions:

How are a function and its inverse related?

Learning Goals:

- I can find inverses of relations.
- I can explore inverses of functions.
- I can find inverses of functions algebraically.
- I can find inverses of nonlinear functions.

An inverse function switches the input and output values.

Inverse functions are functions that "undo" each other.

I can find inverses of relations

Example 1: Finding Inverses of Relations

1) (-2, 6) (1, 5) (4, 4) (7, 3) (10, 2)

(6, -2) (5, 1) (4, 4) (3, 7) (2, 10) (inverse)

2)

Input	0	2	4	6	8	10
Output	0	2	4	4	2	0

Input	0	2	4	4	2	0
Output	0	2	4	6	8	10

You-Try:

1) (-3, -4) (-2, 0) (-1, 4) (0, 8) (1, 12) (2, 16)

(-4, -3) (0, -2) (4, -1) (8, 0) (12, 1) (16, 2)

2)

Input	-2	-1	0	1	2
Output	4	1	0	1	4

Input	4	1	0	1	4
Output	-2	-1	0	1	2

I can explore inverses of functions

Example 2:

1) Let $f(x) = 3x - 5$. Solve $y = f(x)$ for x. Then find the input when the output is 4.

$y = 3x - 5$

$\frac{y+5}{3} = \frac{3x}{3}$

$x = \frac{y+5}{3}$

$x = \frac{4+5}{3}$

$x = \frac{9}{3}$

$\boxed{x = 3}$

The input is 3 when the output is 4

10.4 Inverse of a Function

2) $f(x) = \frac{1}{2}x + 3$

$y = \frac{1}{2}x + 3$

$y - 3 = \frac{1}{2}x$

$\boxed{2(y-3) = x}$

output = 4

$2(4-3) = x$

$2(1) = x$

$\boxed{2 = x}$

3) $f(x) = 4x^2$

$y = 4x^2$

$\frac{y}{4} = x^2$

$\boxed{x = \pm\sqrt{\frac{y}{4}}}$

Output = 4

$x = \pm\sqrt{\frac{4}{4}}$

$x = \pm\sqrt{1}$

$\boxed{x = \pm 1}$

You-Try:

1) Solve $y = f(x)$ for x. Then find the input when the <u>output is</u> 2 for: $f(x) = 2x - 3$

$y = 2x - 3$

$\frac{y+3}{2} = \frac{2x}{2}$

$\frac{y+3}{2} = x$

$\frac{(2+3)}{2} = x$

$\boxed{\frac{5}{2} = x}$

<u>I can find inverses of functions algebraically</u>

Writing a Formula for the Input of a Function

Step 1: Let $y = f(x)$ ⇒ Change $f(x)$ to y

Step 2: switch x and y

Step 3: Solve for y

Step 4: Find the input, if asked

Domain and Range are <u>also switched</u>.

<u>Example 3:</u> Finding the Inverse of a Linear Function

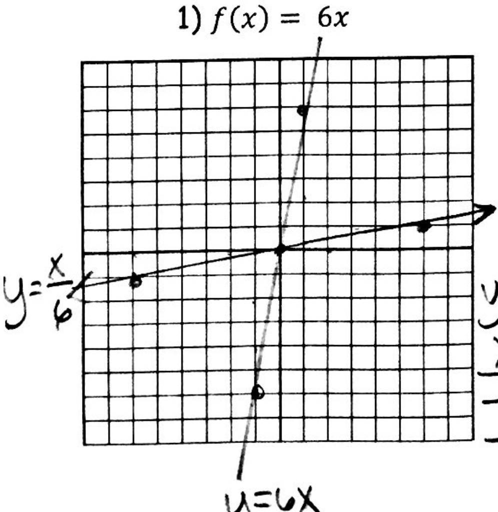
Find the inverse of the following linear functions. Then graph both functions.

1) $f(x) = 6x$

$y = 6x$ (original)

$\frac{x}{6} = \frac{6y}{6}$

$y = \frac{x}{6}$ (inverse)



$y=6x$

x	y
-2	-12
-1	-6
0	0
1	6
2	12

$y = \frac{x}{6}$

x	y
-6	-1
-3	-.5
0	0
3	.5
6	1

2) $f(x) = 3x + 4$

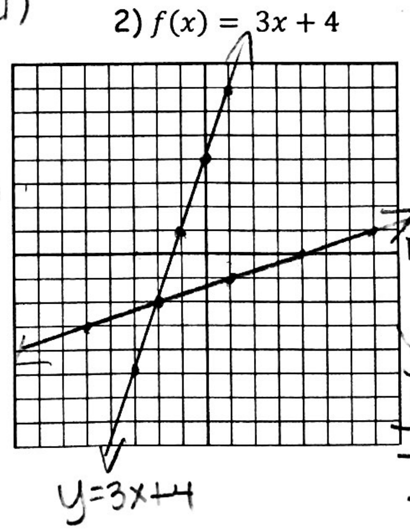
(original)

$y = 3x + 4$

$x = 3y + 4$

$\frac{x-4}{3} = \frac{3y}{3}$

$y = \frac{x-4}{3}$



$y=3x+4$

x	y
-3	-5
-2	-2
-1	1
0	4
1	7

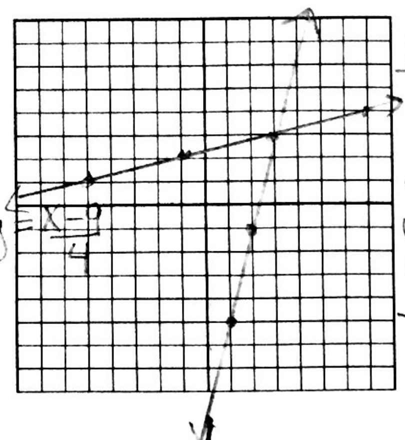
$y = \frac{x-4}{3}$

x	y
-5	-3
-2	-2
1	-1
4	0
7	1

10.4 Inverse of a Function

You-Try:

1) $f(x) = 4x - 9$



$y = \frac{x-9}{4}$

$y = 4x - 9$

$y = 4x - 9$ (orig)

$x = 4y - 9$

$\frac{x+9}{4} = \frac{4y}{4}$

$y = \frac{x+9}{4}$

$y = 4x - 9$

x	y
0	-9
1	-5
2	-1
3	3

$y = \frac{x-9}{4}$

x	y
-5	1
-1	2
3	3
7	4

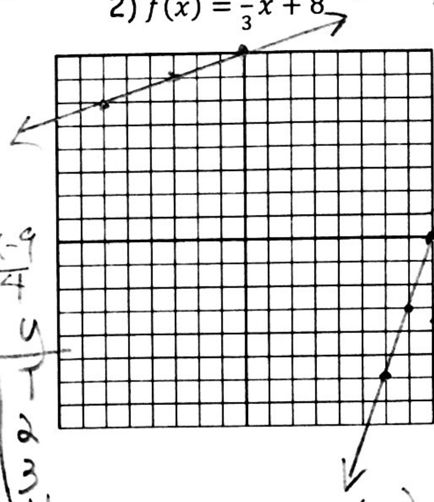
2) $f(x) = \frac{1}{3}x + 8$

$y = \frac{1}{3}x + 8$

$x = \frac{1}{3}y + 8$

$x - 8 = \frac{1}{3}y$

$3(x-8) = y$



$y = 3(x-8)$

$y = \frac{1}{3}x + 8$

x	y
-6	6
-3	7
0	8
3	9

$y = 3(x-8)$

x	y
6	-6
7	-3
8	0
9	3

<u>I can find inverses of nonlinear functions</u>

<u>Example 4:</u> Finding the Inverse of a Quadratic Function

Find the inverse of the following quadratic functions. Then graph both functions.

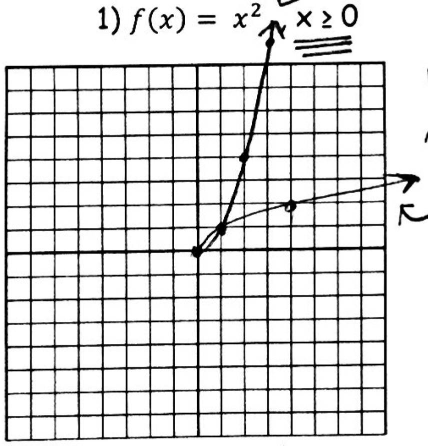
1) $f(x) = x^2, x \geq 0$

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$\sqrt{x} = y$

$y = \sqrt{x}$



$y = x^2 \ (x \geq 0)$

x	y
0	0
1	1
2	4
3	9

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

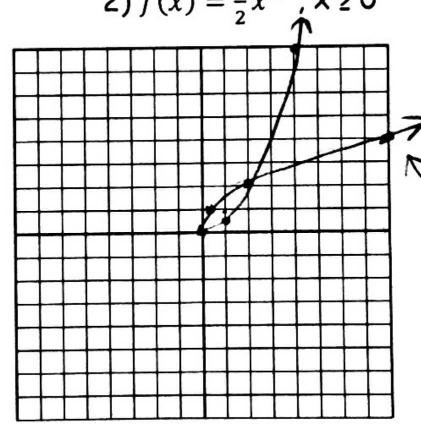
2) $f(x) = \frac{1}{2}x^2, x \geq 0$

$y = \frac{1}{2}x^2$

$x = \frac{1}{2}y^2$

$\sqrt{2x} = \sqrt{y^2}$

$y = \sqrt{2x}$



$y = \frac{1}{2}x^2$

x	y
0	0
1	$\frac{1}{2}$
2	2
4	8

$y = \sqrt{2x}$

x	y
0	0
$\frac{1}{2}$	1
2	2
8	4

10.4 Inverse of a Function

You-Try:

1) $f(x) = -x^2 + 10$, $x \leq 0$

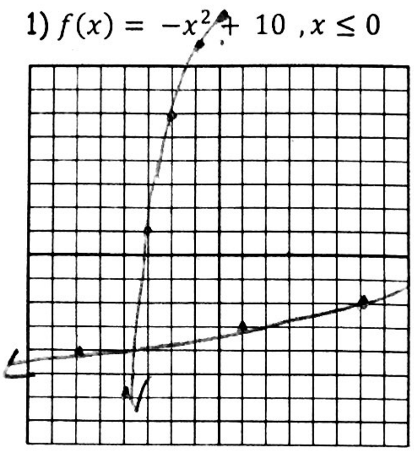
$y = -x^2 + 10$ (org)

$x = -y^2 + 10$

$x - 10 = -y^2$

$10 - x = y^2$

$\sqrt{10 - x} = y$



$y = -x^2 + 10$

x	y
0	10
-1	9
-2	6
-3	1

$y = \sqrt{10 - x}$

x	y
10	0
9	-1
6	-2
1	-3
-6	-4

2) $f(x) = 2x^2 + 6, x \geq 0$

$y = 2x^2 + 6$ $(x \geq 0)$

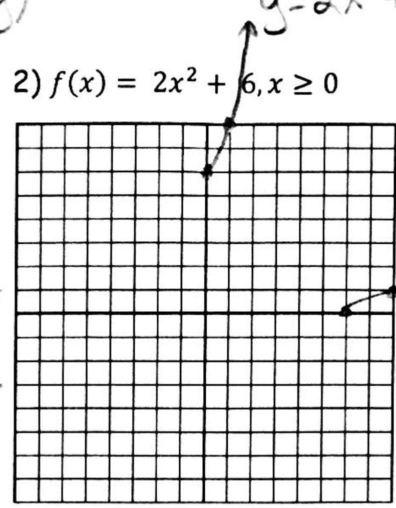
$y = 2x^2 + 6$

$x = 2y^2 + 6$

$\frac{x-6}{2} = \frac{2y^2}{2}$

$y^2 = \frac{x-6}{2}$

$y = \sqrt{\frac{x-6}{2}}$



org

x	y
0	6
1	8
2	14

inverse

x	y
6	0
8	1
14	2

Horizontal Line Test: The inverse of a function f is also a function if and only if no horizontal line intersects the graph of the original function more than once.

Example 5: Finding the Inverse of a Radical Function

Find the inverse of the function. Then graph the function and its inverse to deterime if the inverse is a function.

1) $f(x) = \sqrt{x + 2}$

$x \geq -2, y \geq 0$

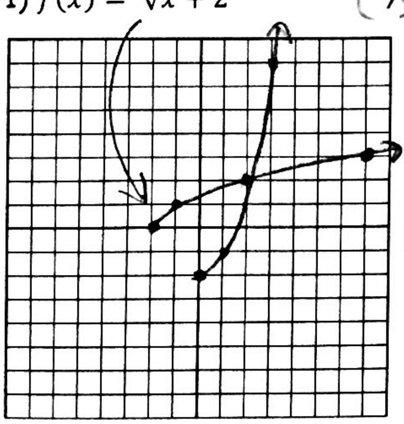
$y = \sqrt{x+2}$

$(x = \sqrt{y+2})^2$

$x^2 = y + 2$

$y = x^2 - 2$

$x \geq 0, y \geq -2$



$y = \sqrt{x+2}$

x	y
-2	0
-1	1
2	2
7	3

$y = x^2 - 2$

$x \geq 0$

x	y
0	-2
1	-1
2	2
3	7

You-Try: $f(x) = \sqrt{2x - 1}$

$x \geq \frac{1}{2}$ $y \geq 0$

$y = \sqrt{2x - 1}$

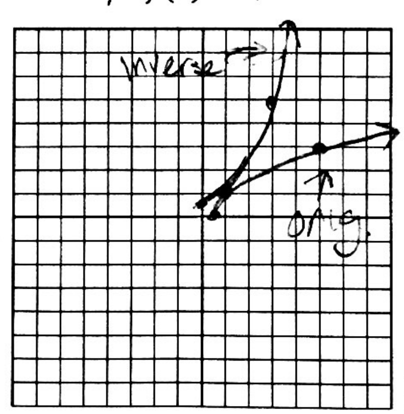
$x = \sqrt{2y - 1}$

$x^2 = 2y - 1$

$x^2 + 1 = 2y$

$\frac{x^2 + 1}{2} = y$

$x \geq 0$ $y \geq \frac{1}{2}$



org

x	y
$\frac{1}{2}$	0
1	1
5	3

inverse

x	y
0	$\frac{1}{2}$
1	1
3	5

Closure: What I learned today was.....