

## 7.4 Solving Polynomials in Factored Form

### Learning Target:

- I can solve a quadratic equation.

### Success Criteria:

- I can use the Zero-Product Property.
- I can factor polynomials using the GCF.
- I can use the Zero-Product Property to solve real-life problems.

### I can use the zero-product property.

A polynomial is in factored form when it is written as a product of factors

Standard Form

$$2x^2 + 5x + 2$$

Factored Form

$$(x+2)(2x+1)$$

\*Use the Zero-Product Property when: one side of equation is in factored form (parentheses) + other side is equal to 0

The solutions of a polynomial equation are called: roots

Examples: Solve each equation.

1)  $3x(x-6) = 0$

$$\frac{3x}{3} = \frac{0}{3}$$

$$\boxed{x=0}$$

$$x - 6 = 0$$

$$+6 \quad +6$$

$$\boxed{x=6}$$

2)  $(x+5)(x-4) = 0$

$$x + 5 = 0$$

$$-5 \quad -5$$

$$\boxed{x=-5}$$

$$x - 4 = 0$$

$$+4 \quad +4$$

$$\boxed{x=4}$$

You-Try: Solve each equation.

1)  $x(x-1) = 0$

$$\boxed{x=0}$$

$$x - 1 = 0$$

$$+1 \quad +1$$

$$\boxed{x=1}$$

2)  $(z-6)(z-8) = 0$

$$z - 6 = 0$$

$$+6 \quad +6$$

$$\boxed{z=6}$$

$$z - 8 = 0$$

$$+8 \quad +8$$

$$\boxed{z=8}$$

## 7.4 Solving Polynomials in Factored Form

When two or more roots of an equation are the same number, the equation has repeated roots.

Examples: Solve each equation.

1)  $(4x + 5)(4x - 5) = 0$

$$\begin{array}{r} 4x+5=0 \\ -5 \quad -5 \end{array} \quad \begin{array}{r} 4x-5=0 \\ +5 \quad +5 \end{array}$$

$$\begin{array}{r} 4x=-5 \\ 4 \quad 4 \end{array} \quad \begin{array}{r} 4x=5 \\ 4 \quad 4 \end{array}$$

$$\boxed{x = -1.25} \quad \boxed{x = 1.25}$$

2)  $(c + 6)^2 = 0$

$$(c+6)(c+6) = 0$$

$$\begin{array}{r} c+6=0 \\ -6 \quad -6 \end{array} \quad \begin{array}{r} c+6=0 \\ -6 \quad -6 \end{array}$$

$$\boxed{c = -6} \quad c = -6$$

repeated roots

You-Try: Solve each equation.

1)  $(a + 5)(a - 2)(a - 7) = 0$

$$\begin{array}{r} a+5=0 \\ -5 \quad -5 \end{array} \quad \begin{array}{r} a-2=0 \\ +2 \quad +2 \end{array} \quad \begin{array}{r} a-7=0 \\ +7 \quad +7 \end{array}$$

$$\boxed{a = -5} \quad \boxed{a = 2} \quad \boxed{a = 7}$$

2)  $(b + 3)^3 = 0$

$$(b+3)(b+3)(b+3) = 0$$

$$\begin{array}{r} b+3=0 \\ -3 \quad -3 \end{array}$$

$$\boxed{b = -3} \text{ repeated root}$$

I can factor polynomials equations using the GCF.

To solve a polynomial equation using the Zero-Product Property, you may need to factor the polynomial, or write it as a product of polynomials. Look for the GCF or greatest common factor of the terms of the polynomial. (This is a monomial that divides evenly into each term.)

reverse distributive

\*Review: Greatest Common Factor = highest # they can be divided by

& variables in common

What is the greatest number that you can divide each set of terms by?

1) 8, 4, 48

$$\boxed{4}$$

2) 3, 9, 54

$$\boxed{3}$$

3) 4, 3, 16

$$\boxed{1}$$

4)  $x, x^2$

$$\boxed{x}$$

5)  $2x, 8x^2, 12x^3$

$$\boxed{2x}$$

6)  $x^5, x^3, x^2$

$$\boxed{x^2}$$

List the greatest common factor of the following terms:

1)  $8x + 16$

$$8(x+2)$$

2)  $x + x^2$

$$x(1+x)$$

3)  $4x^3 + 8x^2 + 16x$

$$4x(x^2 + 2x + 4)$$

## 7.4 Solving Polynomials in Factored Form

Examples: Solve the following equations by factoring out the GCF.

1)  $4x^2 + 12x = 0$

$$4x(x+3) = 0$$

$$\begin{array}{l} \cancel{4}x = 0 \quad x+3 = 0 \\ \cancel{4} \quad \cancel{4} \quad \cancel{+3} \quad \cancel{-3} \end{array}$$

$$x = 0$$

$$x = -3$$

2)  $-10a^2 = 8a$

$$+10a^2 + 8a = 0$$

$$10a^2 + 8a = 0$$

$$2a(5a+4) = 0$$

$$\begin{array}{l} 2a = 0 \quad 5a+4 = 0 \\ \cancel{2} \quad \cancel{2} \quad \cancel{5a} \quad \cancel{+4} \end{array}$$

$$a = 0$$

$$a = -0.8$$

3)  $10x^3 = 15x^2$

$$-15x^2 - 15x^2 = 0$$

$$10x^3 - 15x^2 = 0$$

$$5x^2(2x-3) = 0$$

$$5x^2 = 0 \quad 2x-3 = 0$$

$$x^2 = 0 \quad 2x = 3$$

$$x = \sqrt{0}$$

$$x = 1.5$$

You-Try: Solve the following equations.

1)  $a^2 + 7a = 0$

$$a(a+7) = 0$$

$$a = 0$$

$$a+7 = 0$$

$$a = -7$$

2)  $3s^2 - 9s = 0$

$$3s(s-3) = 0$$

$$\begin{array}{l} 3s = 0 \quad s-3 = 0 \\ \cancel{3} \quad \cancel{3} \quad \cancel{-3} \quad \cancel{+3} \end{array}$$

$$s = 0$$

$$s = 3$$

3)  $4x^2 = 2x$

$$-2x - 2x = 0$$

$$4x^2 - 2x = 0$$

$$2x(2x-1) = 0$$

$$\begin{array}{l} 2x = 0 \quad 2x-1 = 0 \\ \cancel{2} \quad \cancel{2} \quad \cancel{2x} \quad \cancel{-1} \end{array}$$

$$x = 0$$

$$x = 0.5$$

I can use the zero-product property to solve real-life problems.

You can model the arch of an entrance to a train tunnel by using the equation  $y = -\frac{5}{16}(x+8)(x-8)$ , where  $x$  and  $y$  are measured in feet. The  $x$ -axis represents the ground. Find the width of the entrance at ground level.

$$0 = -\frac{5}{16}(x+8)(x-8)$$

$$\begin{array}{l} x+8 = 0 \quad x-8 = 0 \\ \cancel{-8} \quad \cancel{-8} \quad \cancel{+8} \quad \cancel{+8} \end{array}$$

$$x = -8$$

$$x = 8$$

$$16 \text{ ft}$$



You-Try: You can model the entrance to a mine shaft using the equation  $y = -\frac{1}{2}(x+4)(x-4)$ , where  $x$  and  $y$  are measured in feet. The  $x$ -axis represents the ground. Find the width of the entrance at ground level.

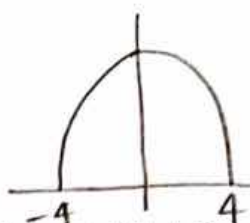
$$0 = -\frac{1}{2}(x+4)(x-4)$$

$$x+4 = 0 \quad x-4 = 0$$

$$x = -4$$

$$x = 4$$

$$8 \text{ ft}$$



Closure: What I learned today was...



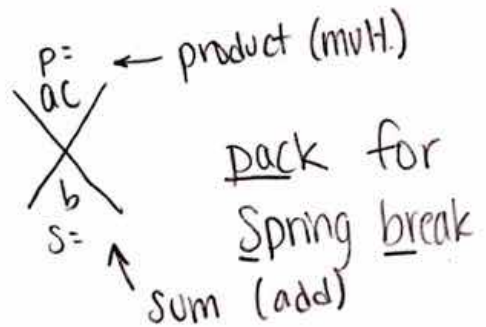
## 7.5 Factoring $x^2 + bx + c = 0$

### Learning Target:

- I can solve a quadratic equation.

### Success Criteria:

- I can factor  $x^2 + bx + c$ .
- I can use factoring to solve real-life problems.



### I can factor $ax^2 + bx + c$ .

Writing a polynomial as a product of factors is called factoring.

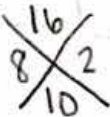
To factor  $ax^2 + bx + c$ , you need to find  $p$  and  $q$  such that  $p+q=b$  and  $p \cdot q=c$ .  
 $(x+p)(x+q)$

Before you can factor an equation, the equation must be equal to 0.

### Factoring $x^2 + bx + c$ , when $b$ and $c$ are positive

Example: Factor each polynomial.

1)  $x^2 + 10x + 16$   
 $a=1$   $b=10$   $c=16$



$(x+8)(x+2)$

2)  $x^2 + 7x + 6 = 0$   
 $a=1$   $b=7$   $c=6$



$(x+6)(x+1) = 0$

$x+6=0$      $x+1=0$   
 $-6$      $-1$

$x = -6$      $x = -1$

You-Try: Factor the polynomials.

1)  $x^2 + 9x + 8 = 0$   
 $a=1$   $b=9$   $c=8$



$(x+8)(x+1) = 0$

$x+8=0$      $x+1=0$   
 $-8$      $-1$

$x = -8$      $x = -1$

2)  $x^2 + 9x + 14$   
 $a=1$   $b=9$   $c=14$



$(x+7)(x+2)$

## 7.5 Factoring $x^2 + bx + c = 0$

**Factoring  $x^2 + bx + c$  when  $b$  is negative and  $c$  is positive**

Example: Factor the polynomial.

1)  $x^2 - 8x + 12 = 0$

$a=1$   $b=-8$   $c=12$

$$\begin{array}{r} 12 \\ -6 \quad -2 \\ \hline -8 \end{array}$$

$(x-6)(x-2) = 0$

$x-6=0$     $x-2=0$

$x=6$

$x=2$

You-Try: Factor each polynomial.

1)  $x^2 + 35 = 12x$

$-12x$     $-12x$

$x^2 - 12x + 35 = 0$

$a=1$   $b=-12$   $c=35$

$(x-7)(x-5) = 0$

$x-7=0$     $x-5=0$

$x=7$

$x=5$

$$\begin{array}{r} 35 \\ -7 \quad -5 \\ \hline -12 \end{array}$$

**Factoring  $x^2 + bx + c$  when  $c$  is negative**

Example: Factor the polynomial.

1)  $4x - 21 = -x^2$

$+x^2$     $+x^2$

$x^2 + 4x - 21 = 0$

$a=1$   $b=4$   $c=-21$

$(x+7)(x-3) = 0$

$x+7=0$     $x-3=0$

$x=-7$

$x=3$

$$\begin{array}{r} -21 \\ 7 \quad -3 \\ \hline 4 \end{array}$$

You-Try: Factor each polynomial.

1)  $x^2 + 2x - 15 = 0$     $a=1$   $b=2$   $c=-15$

$(x+5)(x-3) = 0$

$x+5=0$     $x-3=0$

$x=-5$

$x=3$

$$\begin{array}{r} -15 \\ 5 \quad -3 \\ \hline 2 \end{array}$$

2)  $x^2 - x = 42$

$-42$     $-42$

$x^2 - x - 42 = 0$

$a=1$   $b=-1$   $c=-42$

$(x-7)(x+6) = 0$

$x-7=0$     $x+6=0$

$x=7$

$x=-6$

$$\begin{array}{r} -42 \\ -7 \quad 6 \\ \hline -1 \end{array}$$

$y_1 = ac/x$

$y_2 = ac/x+x$

always top #

Table → last column for  $b$

2)  $x^2 - 13x = -36$

$+36$     $+36$

$x^2 - 13x + 36 = 0$

$a=1$   $b=-13$   $c=36$

$(x-9)(x-4) = 0$

$x-9=0$     $x-4=0$

$x=9$

$x=4$

$$\begin{array}{r} 36 \\ -9 \quad -4 \\ \hline -13 \end{array}$$

2)  $x^2 - 14x + 36 = 12$

$-12$     $-12$

$x^2 - 14x + 24 = 0$

$a=1$   $b=-14$   $c=24$

$(x-12)(x-2) = 0$

$x-12=0$     $x-2=0$

$x=12$

$x=2$

$$\begin{array}{r} 24 \\ -12 \quad -2 \\ \hline -14 \end{array}$$

## 7.5 Factoring $x^2 + bx + c = 0$

### Factoring out the GCF

This should always be considered first when trying to factor any polynomial - it will only become easier.

Example: Factoring out the GCF first, then factor.

$$\begin{aligned}
 1) \quad & 5x^2 + 15x = -10 \\
 & \quad \quad +10 +10 \\
 & 5x^2 + 15x + 10 = 0 \\
 & 5(x^2 + 3x + 2) = 0 \quad a=1 \quad b=3 \quad c=2 \\
 & 5(x+2)(x+1) = 0 \\
 & \quad \quad \boxed{x+2=0} \quad \boxed{x+1=0} \\
 & \quad \quad \boxed{x=-2} \quad \boxed{x=-1}
 \end{aligned}$$

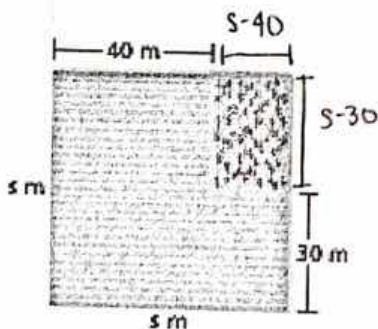
$$\begin{array}{r}
 2/1 \\
 \hline
 2/3
 \end{array}$$

You-Try: Factor each polynomial.

$$\begin{aligned}
 1) \quad & 4x^2 + 32x + 60 = 0 \quad a=1 \quad b=8 \quad c=15 \\
 & 4(x^2 + 8x + 15) = 0 \\
 & 4(x+5)(x+3) = 0 \\
 & \quad \quad \boxed{x+5=0} \quad \boxed{x+3=0} \\
 & \quad \quad \boxed{x=-5} \quad \boxed{x=-3} \\
 & \quad \quad \begin{array}{r} 15/3 \\ \hline 5/8 \end{array} \\
 2) \quad & 3x^2 + 81 = 36x \\
 & \quad \quad -36x -36x \\
 & 3x^2 - 36x + 81 = 0 \\
 & 3(x^2 - 12x + 27) = 0 \\
 & 3(x-9)(x-3) = 0 \\
 & \quad \quad \boxed{x-9=0} \quad \boxed{x-3=0} \\
 & \quad \quad \boxed{x=9} \quad \boxed{x=3} \\
 & \quad \quad \begin{array}{r} 27 \\ \hline -9 \quad -3 \\ -12 \end{array} \quad a=1 \quad b=-12 \quad c=27
 \end{aligned}$$

### I can use factoring to solve real-life problems.

A farmer wants a rectangular pumpkin patch in the northeast corner of a square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?



$$\begin{aligned}
 (s-40)(s-30) &= 600 \\
 s^2 - 30s - 40s + 1200 &= 600 \\
 s^2 - 70s + 1200 &= 600 \\
 \quad \quad -600 \quad -600 \\
 s^2 - 70s + 600 &= 0 \\
 (s-60)(s-10) &= 0
 \end{aligned}$$

$$\begin{aligned}
 a=1 \quad b=-70 \quad c=600 \\
 \begin{array}{r} 600 \\ \hline -60 \quad -10 \\ -70 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 s-60 &= 0 & s-10 &= 0 \\
 s &= 60 & \cancel{s} &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= 60 \cdot 60 \\
 &= \boxed{3600 \text{ m}^2}
 \end{aligned}$$



## 7.6 Factoring $ax^2 + bx + c = 0$

### Learning Target:

- I can solve a quadratic equation.

### Success Criteria:

- I can factor  $ax^2 + bx + c$ .
- I can use factoring to solve real-life problems.



### Factoring $ax^2 + bx + c$ , when $ac$ is Positive Check for GCF first

Example: Factor each polynomial.

1)  $4x^2 + 13x + 3 = 0$      $a=4$     $b=13$     $c=3$

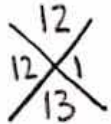
$$\left(\frac{4x+12}{4}\right)\left(\frac{4x+1}{4}\right) = 0$$

$$(x+3)(4x+1) = 0$$

$$x+3=0 \quad 4x+1=0$$

$$\boxed{x = -3}$$

$$\boxed{x = -0.25}$$



You-Try: Factor the polynomials.

1)  $4x^2 + 12 = -14x$   
 $\quad \quad \quad +14x \quad +14x$

$$4x^2 + 14x + 12 = 0$$

$$2(2x^2 + 7x + 6) = 0$$

$$2\left(\frac{2x+4}{2}\right)\left(\frac{2x+3}{2}\right) = 0$$

$$2(x+2)(2x+3) = 0$$

$$x+2=0$$

$$2x+3=0$$

$$\boxed{x = -2}$$

$$2x = -3$$

$$\boxed{x = -1.5}$$

$a=2$     $b=7$     $c=6$



### Factoring $ax^2 + bx + c$ when $ac$ is Negative

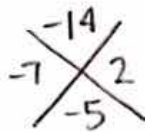
Example: Factor the polynomial.

$a=2$     $b=-5$     $c=-7$

1)  $2x^2 - 5x - 7$

$$(2x-7)\left(\frac{2x+2}{2}\right)$$

$$\boxed{(2x-7)(x+1)}$$



2)  $9x^2 - 21x = -6$   
 $\quad \quad \quad +6 \quad +6$

$$9x^2 - 21x + 6 = 0$$

$$3(3x^2 - 7x + 2) = 0$$

$$3\left(\frac{3x-6}{3}\right)\left(\frac{3x-1}{3}\right) = 0$$

$$3(x-2)(3x-1) = 0$$

$$x-2=0$$

$$3x-1=0$$

$$\boxed{x = 2}$$

$$3x = 1$$

$$\boxed{x = \frac{1}{3}}$$

2)  $-4x^2 = -7x + 3$   
 $\quad \quad \quad +4x^2 \quad +4x^2$

$$0 = 4x^2 - 7x + 3$$

$$0 = \left(\frac{4x-4}{4}\right)(4x-3)$$

$$0 = (x-1)(4x-3)$$

$$x-1=0$$

$$4x-3=0$$

$$\boxed{x = 1}$$

$$4x = 3$$

$$\boxed{x = \frac{3}{4}}$$

$a=3$     $b=-7$     $c=2$



$a=4$     $b=-7$     $c=3$



## 7.6 Factoring $ax^2 + bx + c = 0$

### Factoring $ax^2 + bx + c$ when $a$ is Negative

Example: Factor the polynomial.

$$1) -4x^2 - 8x = -5$$

$$-4x^2 - 8x + 5 = 0$$

$$-1(4x^2 + 8x - 5) = 0$$

$$-1\left(\frac{4x+10}{2}\right)\left(\frac{4x-2}{2}\right) = 0$$

$$-1(2x+5)(2x-1) = 0$$

$$\begin{array}{r} -20 \\ +10 \quad -2 \\ +8 \end{array}$$

$$2x+5=0$$

$$2x=-5$$

$$x = -\frac{5}{2}$$

$$2x-1=0$$

$$2x=1$$

$$x = \frac{1}{2}$$

You-Try: Factor each polynomial.

$$1) -2y^2 - 5y - 3 \quad a=2 \quad b=5$$

$$-1(2y^2 + 5y + 3) \quad c=3$$

$$-1\left(\frac{2y+2}{2}\right)\left(\frac{2y+3}{2}\right)$$

$$-1(y+1)(2y+3)$$

$$\begin{array}{r} 6 \\ 2 \times 3 \\ 5 \end{array}$$

$$2) 31x + 15 = -14x^2$$

$$+14x^2$$

$$14x^2 + 31x + 15 = 0$$

$$\left(\frac{14x+21}{7}\right)\left(\frac{14x+10}{2}\right) = 0$$

$$(2x+3)(7x+5) = 0$$

$$2x+3=0 \quad 7x+5=0$$

$$2x=-3$$

$$x = -\frac{3}{2}$$

$$7x=-5$$

$$x = -\frac{5}{7}$$

$$\begin{array}{r} 210 \\ 10 \quad 21 \\ 31 \end{array}$$

$$3) 2x^2 + 5 = 7x$$

$$-7x \quad -7x$$

$$2x^2 - 7x + 5 = 0$$

$$\left(\frac{2x-5}{2}\right)\left(\frac{2x-2}{2}\right) = 0$$

$$(2x-5)(x-1) = 0$$

$$2x-5=0$$

$$2x=5$$

$$x = \frac{5}{2}$$

$$x-1=0$$

$$x=1$$

$$a=2 \quad b=-7$$

$$c=5$$

$$\begin{array}{r} 10 \\ -5 \quad -2 \\ -1 \end{array}$$

I can use factoring to solve real-life problems.

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. What is the width of the reserve?



$$2W+1$$

$$W(2W+1) = 55$$

$$2W^2 + W = 55$$

$$2W^2 + W - 55 = 0$$

$$\left(\frac{2W+11}{2}\right)\left(\frac{2W-10}{2}\right) = 0$$

$$(2W+11)(W-5) = 0$$

$$2W+11=0 \quad W-5=0$$

$$2W=-11$$

$$W=5$$

$$W = -5.5$$

$$\begin{array}{r} -110 \\ 11 \quad -10 \\ 1 \end{array}$$



## 7.7 Factoring Special Products

### Learning Target:

- I can solve a quadratic equation.

### Success Criteria:

- I can factor the difference of two squares.
- I can factor perfect square trinomials.
- I can use factoring to solve real-life problems.

### I can factor the difference of two squares.

$$\begin{array}{l} x^2 - 81 \\ x^2 + 0x - 81 \\ (x+9)(x-9) \end{array}$$

$$\begin{array}{r} 81 \\ 9 \times 9 \\ \hline 0 \end{array}$$

Difference of Two Squares Pattern:

$$a^2 - b^2 = (a-b)(a+b)$$

Example: Factor the polynomials.

1)  $x^2 - 64$

$$(x-8)(x+8)$$

2)  $25b^2 - 36$

$$(5b-6)(5b+6)$$

You-Try: Factor the polynomial.

1)  $100 - m^2$

$$(10-m)(10+m)$$

2)  $16b^2 - 49$

$$(4b-7)(4b+7)$$

### I can factor perfect square trinomials.

$$\begin{array}{l} x^2 + 6x + 9 \\ (x+3)(x+3) \\ (x+3)^2 \end{array}$$

$$\begin{array}{r} 9 \\ 3 \times 3 \\ \hline 6 \end{array}$$

Perfect Square Trinomial Pattern

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

## 7.7 Factoring Special Products

Example: Factor each polynomial.

$$1) n^2 + 8n + 16$$
$$\boxed{(n+4)^2}$$

$$2) 4x^2 - 12x + 9$$
$$\boxed{(2x-3)^2}$$

You-Try: Factor each polynomial.

$$1) m^2 - 2m + 1$$
$$\boxed{(m-1)^2}$$

$$2) 9z^2 + 36z + 36$$
$$\boxed{(3z+6)^2}$$

### Solving a Polynomial Equation

Example: Solve the polynomial equation.

$$1) x^2 + 3x + \frac{9}{4} = 0$$

$$\left(x + \frac{3}{2}\right)^2 = 0$$

$$x + \frac{3}{2} = 0$$

$$\boxed{x = -\frac{3}{2}}$$

$$2) n^2 - 81 = 0$$

$$(n-9)(n+9) = 0$$

$$\boxed{n=9}$$

$$\boxed{n=-9}$$

You-Try: Solve the polynomial equation.

$$1) a^2 + 6a + 9 = 0$$

$$(a+3)(a+3) = 0$$

$$\boxed{a=-3}$$

## 7.7 Factoring Special Products

I can use factoring to solve real-life problems.

A bird picks up a golf ball and drops it while flying. The function represents the height  $y$  (in feet) of the golf ball  $t$  seconds after it is dropped. The ball hits the top of a 32-foot-tall pine tree. After how many seconds does the ball hit the tree?



$$\begin{array}{r} 32 = 81 - 16t^2 \\ -32 \quad -32 \\ \hline \end{array}$$

$$0 = 49 - 16t^2$$

$$0 = (7 - 4t)(7 + 4t)$$

$$\begin{array}{l} 7 - 4t = 0 \quad 7 + 4t = 0 \\ -4t = -7 \quad 4t = -7 \end{array}$$

$$\boxed{t = 1.75}$$

$$\cancel{t = 1.75}$$

**You-Try:** What if the golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

$$0 = 81 - 16t^2$$

$$0 = (9 - 4t)(9 + 4t)$$

$$0 = 9 - 4t \quad 0 = 9 + 4t$$

$$-9 = -4t \quad -9 = 4t$$

$$\boxed{t = 2.25}$$

$$\cancel{t = 2.25}$$



## 7.8 Factoring Polynomials Completely

### Learning Target:

- I can solve a quadratic equation.

### Success Criteria:

- I can factor polynomials by grouping.
- I can factor polynomials completely.
- I can use factoring to solve real-life problems.

### I can factor polynomials by grouping.

Used for polynomials with four terms. Factor the GCF out of each pair of terms.  
Look for and factor out the common binomial factor.

Example: Factor each polynomial by grouping.

$$1) [x^3 + 3x^2][+2x + 6]$$

$$x^2(x+3) + 2(x+3)$$

$$(x^2+2)(x+3)$$

$$2) [4x^3 - 16x^2][-8x + 32]$$

$$4x^2(x-4) - 8(x-4)$$

$$(4x^2-8)(x-4)$$

You-Try: Factor each polynomial by grouping.

$$1) [a^3 + 3a^2][a - 3]$$

$$a^2(a+3) - 1(a+3)$$

$$(a^2-1)(a+3)$$

$$2) [y^2 + yx][+2y + 2x]$$

$$y(y+x) + 2(y+x)$$

$$(y+2)(y+x)$$

## 7.8 Factoring Polynomials Completely

I can factor polynomials completely.

4 specific guidelines for Factoring polynomials completely:

|  |   |
|--|---|
| 1. Factor out GCF  | $3x^2 + 6x \rightarrow 3x(x+2)$   |
| 2. Look for patterns (Difference of squares OR perfect square) | $x^2 - 9 \rightarrow (x-3)(x+3)$ OR $\frac{(x^2 - 6x + 9)}{(x-3)^2}$              |
| 3. Factor with $ax^2 + bx + c$ $\frac{ac}{b}$                  | $3x^2 - 5x - 2$ $\frac{-6}{-6 \cdot -5}$ $\frac{3x-6}{(x-2)} \frac{3x+1}{(3x+1)}$ |
| 4. Factor by grouping (4 terms)                                | $x^3 - 4x^2 + x - 4 = x^2(x-4) + 1(x-4)$<br>$(x^2+1)(x-4)$                        |

Example: Factor completely.

1)  $3x^3 + 6x^2 - 18x$

$3x(x^2 + 2x - 6)$

$\frac{6}{2}$

nothing multiplies to -6 and adds to 2  
so completely factored

2)  $7x^4 - 28x^2$

$7x^2(x^2 - 4)$

$7x^2(x-2)(x+2)$

You-Try: Factor completely.

1)  $2x^3 + 6x^2 - 2x$

$2x(x^2 + 3x - 1)$

$\frac{-1}{3}$

2)  $5x^4 - 45x^2$

$5x^2(x^2 - 9)$

$5x^2(x-3)(x+3)$

## 7.8 Factoring Polynomials Completely

### Solving an Equation by Factoring Completely

Example: Solve Completely.

$$1) \quad 2x^3 + 8x^2 = 10x$$

$$-10x - 10x$$

$$2x^3 + 8x^2 - 10x = 0$$

$$2x(x^2 + 4x - 5) = 0$$

$$\frac{-5}{4} - 1$$

$$2x(x+5)(x-1) = 0$$

$$2x = 0 \quad x+5 = 0 \quad x-1 = 0$$

$$\boxed{x=0}$$

$$\boxed{x=-5}$$

$$\boxed{x=1}$$

$$2) \quad 3x^3 + 6x^2 = 24x$$

$$-24x - 24x$$

$$3x^3 + 6x^2 - 24x = 0$$

$$3x(x^2 + 2x - 8) = 0$$

$$\frac{-8}{2} - 2$$

$$3x(x+4)(x-2) = 0$$

$$3x = 0 \quad x+4 = 0 \quad x-2 = 0$$

$$\boxed{x=0}$$

$$\boxed{x=-4}$$

$$\boxed{x=2}$$

You-Try: Solve Completely.

$$1) \quad w^3 - 8w^2 + 16w = 0$$

$$w(w^2 - 8w + 16) = 0$$

$$w(w-4)(w-4) = 0$$

$$\boxed{w=0}$$

$$\boxed{w=4}$$

$$2) \quad c^3 - 7c^2 = -12c$$

$$+12c + 12c$$

$$c^3 - 7c^2 + 12c = 0$$

$$c(c^2 - 7c + 12) = 0$$

$$\frac{12}{-7} - 3$$

$$c(c-4)(c-3) = 0$$

$$\boxed{c=0}$$

$$c-4=0$$

$$c-3=0$$

$$\boxed{c=4}$$

$$\boxed{c=3}$$