9.4 Solving Quadratic Equations by Completing the Square

Learning Targets:

- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can complete the square for expressions of the form $x^2 + bx$
- I can solve quadratic equations by completing the square
- I can find and use maximum and minimum values
- I can solve real-life problems by completing the square

I can complete the square for expressions of the form $x^2 + bx$

Completing the Square: for an expression of the form x^2+bx, you can add a constant c to the expression so that x^2+bx+c is a perfect square trinomial.

Step 1: Find $\frac{1}{2}$ of b, the coefficient of x.

$(x+\#)^2$ or $(x-\#)^2$

Step 2: Square the result of Step 1.

Step 3: Add the result from Step 2 to $x^2 + bx$

Step 4: Factor the trinomial

Example 1: Complete the square for the expression. Then factor the trinomial.

a) $x^2 + 18x$

Step 1: $\frac{18}{2}=9$

Step 2: $9^2=81$

Step 3: $x^2+18x+81$

Step 4: $\boxed{(x+9)^2}$

b) $x^2 - 3x$

Step 1: $\frac{-3}{2}$

Step 2: $\left(-\frac{3}{2}\right)^2=\frac{9}{4}$

Step 3: $x^2-3x+\frac{9}{4}$

Step 4: $\left(x-\frac{3}{2}\right)^2$

You try: $x^2 - 4x$

Step 1: $\frac{-4}{2}=-2$

Step 2: $(-2)^2=4$

Step 3: x^2-4x+4

Step 4: $\boxed{(x-2)^2}$

You try: $x^2 + 14x$

Step 1: $\frac{14}{2}=7$

Step 2: $7^2=49$

Step 3: $x^2+14x+49$

Step 4: $\boxed{(x+7)^2}$

I can solve quadratic equations by completing the square

Solving Quadratic Equations by Completing the Square

The method of Completing the Square can be used to solve any quadratic equation. The equation must be written in the form $x^2+bx=d$. To keep the equation balanced, the result of Step 2 from above must be added to BOTH sides of the equation.

9.4 Solving Quadratic Equations by Completing the Square

Example 2: Solve by Completing the Square.

a) $x^2 - 18x = -17$

Step 1: $\frac{-18}{2} = -9$

Step 2: $(-9)^2 = 81$

$x^2 - 18x + 81 = -17 + 81$

$(x-9)^2 = 64$

$\sqrt{(x-9)^2} = \sqrt{64}$

$x - 9 = \pm 8$

$x-9 = 8$ $\quad$ $x-9=-8$

$\boxed{x=17}$ $\quad$ $\boxed{x=1}$

b) $m^2 + 12m = -8$

Step 1: $\frac{12}{2} = 6$

Step 2: $(6)^2 = 36$

$m^2 + 12m + 36 = -8 + 36$

$(m+6)^2 = 28$

$\sqrt{(m+6)^2} = \sqrt{28}$

$m+6 = \pm\sqrt{28}$

$m+6 = \sqrt{28}$ $\quad$ $m+6 = -\sqrt{28}$

$\boxed{m = -6+\sqrt{28}}$ $\quad$ $\boxed{m=-6-\sqrt{28}}$

You try: $x^2 - 2x = 3$

Step 1: $\frac{-2}{2} = -1$

Step 2: $(-1)^2 = 1$

$x^2 - 2x + 1 = 3 + 1$

$(x-1)^2 = 4$

$\sqrt{(x-1)^2} = \sqrt{4}$

$x - 1 = \pm 2$

$x-1=2$ $\quad$ $x-1=-2$

$\boxed{x=3}$ $\quad$ $\boxed{x=-1}$

c) $x^2 - 12x + 9 = 0$

$\quad\quad -9 \quad -9$

$x^2 - 12x = -9$

Step 1: $\frac{-12}{2} = -6$

Step 2: $(-6)^2 = 36$

$x^2 - 12x + 36 = -9 + 36$

$(x-6)^2 = 25$

$\sqrt{(x-6)^2} = \sqrt{25}$

$x - 6 = \pm 5$

$x-6=5$ $\quad$ $x-6=-5$

$\boxed{x=11}$ $\quad$ $\boxed{x=1}$

d) $\frac{2x^2 + 12x - 10}{2} = \frac{0}{2}$

$x^2 + 6x - 5 = 0$

$x^2 + 6x = 5$

Step 1: $\frac{6}{2} = 3$

Step 2: $3^2 = 9$

$x^2 + 6x + 9 = 5 + 9$

$(x+3)^2 = 14$

$\sqrt{(x+3)^2} = \sqrt{14}$

$x + 3 = \pm\sqrt{14}$

$x+3 = \sqrt{14}$ $\quad$ $x+3 = -\sqrt{14}$

$\boxed{x=-3+\sqrt{14}}$ $\quad$ $\boxed{x=-3-\sqrt{14}}$

You try: $x^2 + 14x - 10 = 0$

$x^2 + 14x = 10$

Step 1: $\frac{14}{2} = 7$

Step 2: $7^2 = 49$

$x^2 + 14x + 49 = 10 + 49$

$(x+7)^2 = 59$

$\sqrt{(x+7)^2} = \sqrt{59}$

$x + 7 = \pm\sqrt{59}$

$x+7 = \sqrt{59}$ $\quad$ $x+7 = -\sqrt{59}$

$\boxed{x=-7+\sqrt{59}}$ $\quad$ $\boxed{x=-7-\sqrt{59}}$

You try: $\frac{3x^2 + 6x - 9}{3} = \frac{0}{3}$

$x^2 + 2x - 3 = 0$

$x^2 + 2x = 3$

Step 1: $\frac{2}{2} = 1$

Step 2: $1^2 = 1$

$x^2 + 2x + 1 = 3 + 1$

$(x+1)^2 = 4$

$\sqrt{(x+1)^2} = \sqrt{4}$

$x + 1 = \pm 2$

$x+1=2$ $\quad$ $x+1=-2$

$\boxed{x=1}$ $\quad$ $\boxed{x=-3}$

I can find and use maximum and minimum values

Recall: The vertex of a quadratic function can be used to find the maximum or minimum value of the function. We can use Completing the Square to write a quadratic in vertex form, $y = a(x-h)^2 + k$, to find the max/min value.

$a \neq 0$

9.4 Solving Quadratic Equations by Completing the Square

Example 3: Find the maximum or minimum value of each quadratic function.

a) $y = x^2 + 8x + 5$

$-5 \qquad -5$

$y - 5 = x^2 + 8x + 16$

$+16$

$y + 11 = (x+4)^2$

$-11 \qquad -11$

$\boxed{y = (x+4)^2 - 11}$

$(-4,-11)$

min value
-11

Max OR Min

b) $y = -x^2 + 4x + 2$

$-2 \qquad -2$

$y - 2 = -(x^2 - 4x + 4)$

-4

$y - 6 = -(x-2)^2$

$+6 \qquad +6$

$y = -(x-2)^2 + 6$

$(2,6)$

max value
6

Max OR Min

You try: $y = x^2 - 10x + 8$

$-8 \qquad -8$

$y - 8 = x^2 - 10x + 25$

$+25$

$y + 17 = (x-5)^2$

$-17 \qquad -17$

$y = (x-5)^2 - 17$

$(5,-17)$

min value
-17

Max OR Min

You try: $y = -x^2 + 8x - 2$

$+2 \qquad +2$

$y + 2 = -(x^2 + 8x + 16)$

$+16$

$y + 18 = -(x+4)^2$

$-18 \qquad -18$

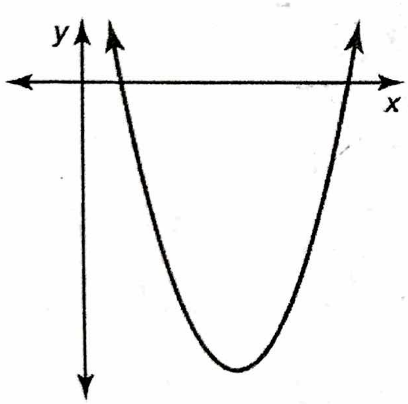
$y = -(x+4)^2 - 18$

$(-4,-18)$

max value
-18

Max OR Min

Example 4: Which of the functions could be represented by the graph? Explain.



~~$f(x) = (x - 4)^2 + 9$~~

no - vertex (4,9)

~~$g(x) = -(x - 3)^2 + 5$~~

no - opens down

~~$m(x) = -\frac{1}{4}(x + 2)(x + 6)$~~

no - opens down

$p(x) = (x - 1)(x - 7)$

zeros at 1 + 7

9.4 Solving Quadratic Equations by Completing the Square

I can solve real-life problems by completing the square

Example 5: The function $y = -16x^2 + 160x$ represents the height y (in feet) of a model rocket x seconds after it is launched.

a) Find the maximum height of the rocket.

$y = -16(x^2 - 10x \underline{+25})$ $\qquad y = -16(x-5)^2 + 400$

-400

$y - 400 = -16(x-5)^2$

$+400 \qquad +400$

$\boxed{\text{max height @ 400 ft}}$

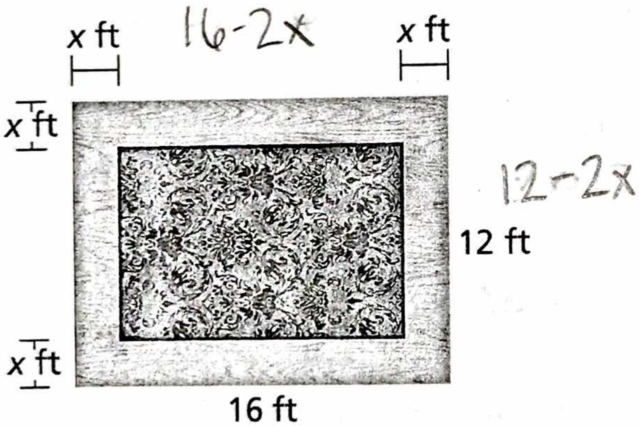
b) Find and interpret the axis of symmetry.

a.o.s. $x = 5$ $\qquad$ At 5 seconds, rocket changes direction

left of a.o.s. rocket increasing

right of a.o.s. rocket decreasing

Example 6: You decide to put a rug on a floor. You want the rug to cover 100 square feet and to have a uniform border of floor visible, as shown. Find the width of the border to the nearest inch.



$100 = (16-2x)(12-2x)$

$100 = 192 - 32x - 24x + 4x^2$

$100 = 4x^2 - 56x + 192$

$-192 \qquad -192$

$-92 = 4x^2 - 56x$

$-92 = 4(x^2 - 14x \underline{+49})$

$+196$

$\frac{104}{4} = \frac{4(x-7)^2}{4}$

$26 = (x-7)^2$

$\pm\sqrt{26} = x - 7$

$x - 7 = \sqrt{26} \qquad x - 7 = -\sqrt{26}$

$x = 7 + \sqrt{26} \qquad x = 7 - \sqrt{26}$

$x \approx 12.1$ ft $\qquad x \approx 1.9$ ft

extraneous $\qquad \boxed{23 \text{ in}}$

Closure: What I learned today was...

9.5 Solving Quadratic Equations Using the Quadratic Formula

Learning Targets:

- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can solve quadratic equations using the Quadratic Formula
- I can interpret the discriminant
- I can choose efficient methods for solving quadratic equations

I can solve quadratic equations using the Quadratic Formula

Quadratic Formula: ax^2+bx+c

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This formula is used to find the real solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, and $b^2-4ac \geq 0$.

Example 1: Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

a) $x^2 + 6x - 13 = 0$ $a=1$ $b=6$ $c=-13$

$$x=\frac{-6\pm\sqrt{6^2-4(1)(-13)}}{2}$$

$$x=\frac{-6\pm\sqrt{36+52}}{2}$$

$$x=\frac{-6\pm\sqrt{88}}{2}$$

$\frac{-6+\sqrt{88}}{2}$ $\boxed{x\approx 7.7}$

$\frac{-6-\sqrt{88}}{2}$ $\boxed{x\approx -1.7}$

b) $-3x^2 + 5x - 1 = -7$ $a=-3$ $b=5$ $c=6$

$-3x^2+5x+6=0$

$$x=\frac{-5\pm\sqrt{5^2-4(-3)(6)}}{2(-3)}$$

$$x=\frac{-5\pm\sqrt{25+72}}{-6}$$

$$x=\frac{-5\pm\sqrt{97}}{-6}$$

$\frac{-5+\sqrt{97}}{-6}$ $\boxed{x\approx -0.8}$

$\frac{-5-\sqrt{97}}{-6}$ $\boxed{x\approx 2.5}$

You try: $x^2 + 2x - 12 = 0$ $a=1$ $b=2$ $c=-12$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-12)}}{2}$$

$$x=\frac{-2\pm\sqrt{4+48}}{2}$$

$$x=\frac{-2\pm\sqrt{52}}{2}$$

$\frac{-2+\sqrt{52}}{2}$ $\boxed{x\approx 2.6}$

$\frac{-2-\sqrt{52}}{2}$ $\boxed{x\approx -4.6}$

You try: $12x^2 - 4x - 5 = 0$ $a=12$ $b=-4$ $c=-5$

$$x=\frac{4\pm\sqrt{(-4)^2-4(12)(-5)}}{2(12)}$$

$$x=\frac{4\pm\sqrt{16+240}}{24}$$

$$x=\frac{4\pm\sqrt{256}}{24}$$

$$x=\frac{4\pm 16}{24}$$

$\frac{4+16}{24}=\frac{20}{24}=\boxed{0.8}$

$\frac{4-16}{24}=\frac{-12}{24}=\boxed{-0.5}$

9.5 Solving Quadratic Equations Using the Quadratic Formula

Example 2: The number *y* of Northern Rocky Mountain wolf breeding pairs *x* years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 77 breeding pairs of wolves?

$$\cancel{77} = 0.2x^2 + 1.8x - 3$$
$$-\cancel{77} \qquad -77$$
$$0 = 0.2x^2 + 1.8x - 80$$
$$a = 0.2 \quad b = 1.8 \quad c = -80$$

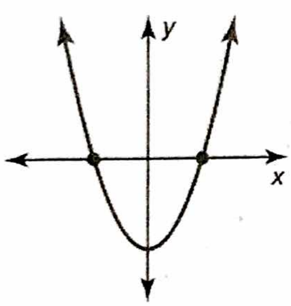
$$\boxed{\begin{matrix}2006 \\ 2030\end{matrix}}$$

$$x = \frac{-1.8 \pm \sqrt{1.8^2 - 4(0.2)(-80)}}{2(0.2)}$$
$$x = \frac{-1.8 \pm \sqrt{3.24 + 64}}{0.4}$$
$$x = \frac{-1.8 \pm \sqrt{67.24}}{0.4}$$
$$x = \frac{-1.8 \pm 8.2}{0.4}$$
$$\nearrow \frac{-1.8 + 8.2}{0.4} = \frac{6.4}{0.4} = 16$$
$$\searrow \frac{-1.8 - 8.2}{0.4} = \frac{-10}{0.4} = 40$$

I can interpret the discriminant

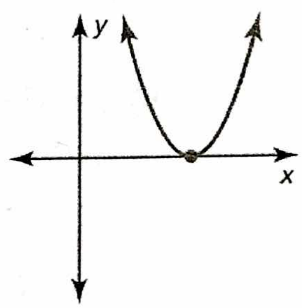
Discriminant: In the Quadratic Formula, the expression $b^2 - 4ac$ is called the discriminant. Since the discriminant is under the square root, its value allows us to see the number of solutions that the quadratic equation has.

$b^2 - 4ac > 0$



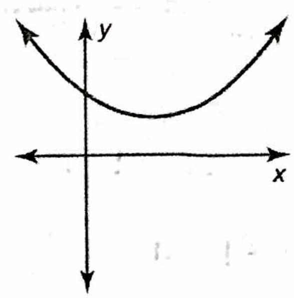
2 real solutions
(2 x-ints)

$b^2 - 4ac = 0$



1 real solution
(1 x-int)

$b^2 - 4ac < 0$



0 real solution
(0 x-int)

9.5 Solving Quadratic Equations Using the Quadratic Formula

Example 3: Determine the number of real solutions of each quadratic.

a) $-x^2 + 6x + 3 = 0$ $a=-1$ $b=6$ $c=3$

$6^2 - 4(-1)(3)$

$36 + 12$

48 $\boxed{\text{2 real solutions}}$

b) $x^2 + 3x + 8 = 0$ $a=1$ $b=3$ $c=8$

$3^2 - 4(1)(8)$

$9 - 32$

-23 $\boxed{\text{no real solutions}}$

You try: $2x^2 - 3x - 1 = 0$ $a=2$ $b=-3$ $c=-1$

$(-3)^2 - 4(2)(-1)$

$9 + 8$

17 $\boxed{\text{2 real solutions}}$

You try: $6x^2 + 2x + 1 = 0$ $a=6$ $b=2$ $c=1$

$2^2 - 4(6)(1)$

$4 - 24$

-20 $\boxed{\text{no real solutions}}$

Example 4: Find the number of x-intercepts of each graph using the discriminant. Check your results by graphing.

a) $y = 2x^2 - 4x - 3$

$a=2$ $b=-4$ $c=-3$

$(-4)^2 - 4(2)(-3)$

$16 + 24$

40

$\boxed{\text{2 real solutions} \therefore \text{2 x-intercepts}}$

b) $y = -x^2 - 8x - 18$

$a=-1$ $b=-8$ $c=-18$

$(-8)^2 - 4(-1)(-18)$

$64 - 72$

-8

$\boxed{\text{no real solutions} \therefore \text{no x-intercepts}}$

9.5 Solving Quadratic Equations Using the Quadratic Formula

I can choose efficient methods for solving quadratic equations

Overview of Methods

Method	Advantages	Disadvantages
Factoring (*Lessons 7.5–7.8*)	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing (*Lesson 9.2*)	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots (*Lesson 9.3*)	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square (*Lesson 9.4*)	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula (*Lesson 9.5*)	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations

Example 5: Solve the equation using any method. Explain your choice of method.

a) $x^2 - 4x - 7 = 0$ completing the square $a=1$ b even

$x^2-4x=7$

$x^2-4x+4=11$

$(x-2)^2=11$

$x-2=\pm\sqrt{11}$

$x-2=\sqrt{11}$ $\boxed{x=2+\sqrt{11}}$

$x-2=-\sqrt{11}$ $\boxed{x=2-\sqrt{11}}$

b) $(x - 2)^2 = 64$ square roots $x^2=d$

$x-2=\pm 8$

$x-2=8$ $\boxed{x=10}$

$x-2=-8$ $\boxed{x=-6}$

c) $10x^2 + 7x - 12 = 0$ quad form large #s

$x=\dfrac{-7\pm\sqrt{7^2-4(10)(-12)}}{20}$

$x=\dfrac{-7\pm\sqrt{49+480}}{20}$

$x=\dfrac{-7\pm\sqrt{529}}{20}$

$x=\dfrac{-7\pm 23}{20}$

$\dfrac{-7+23}{20}=\boxed{0.8}$

$\dfrac{-7-23}{20}=\boxed{-1.5}$

d) $3x^2 + x - 5 = 0$ quad form b not even

$x=\dfrac{-1\pm\sqrt{1^2-4(3)(-5)}}{6}$

$x=\dfrac{-1\pm\sqrt{1+60}}{6}$

$x=\dfrac{-1\pm\sqrt{61}}{6}$

$x=\dfrac{-1+\sqrt{61}}{6}$ $\boxed{x\approx 1.1}$

$x=\dfrac{-1-\sqrt{61}}{6}$ $\boxed{x\approx -1.5}$

Closure: What I learned today was...

9.6 Solving Nonlinear Systems of Equations

Learning Targets:

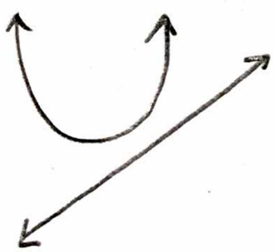
- I can use properties of radicals to simplify, solve, and graph expressions and equations.
- I can use graphs of quadratics to solve.

Success Criteria:

- I can solve systems of nonlinear equations by graphing
- I can solve systems of nonlinear equations algebraically
- I can approximate solutions of nonlinear systems and equations

I can solve systems of nonlinear equations by graphing

System of nonlinear equations: a system in which __at least__ one of the equations is __nonlinear__. When a nonlinear system has a __linear__ equation and a __quadratic__ equation, the graphs can intersect in __zero__, __one__, or __two__ points (solutions).



zero solutions

one solution

two solutions

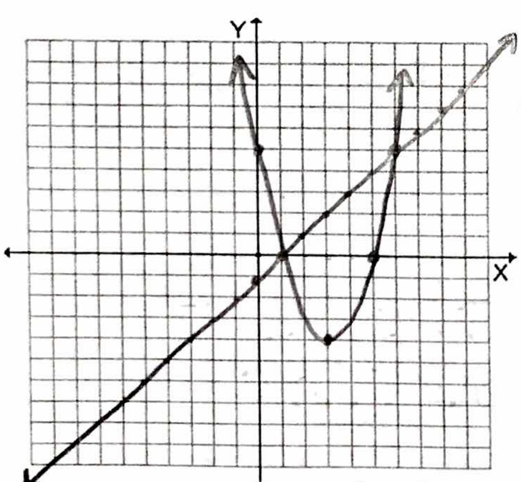
Example 1: Solving the system by graphing.

a) $y = x^2 - 6x + 5$ $(x-5)(x-1)$ vertex $(3,-4)$

$y = x - 1$

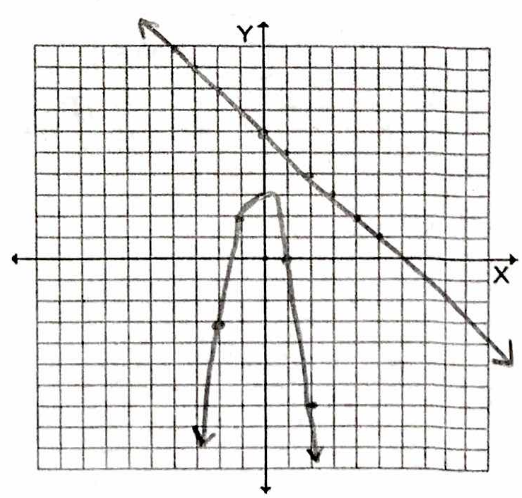
b) $y = -x + 6$

$y = -2x^2 - x + 3$



Solutions: $(1,0)$

$(6,5)$



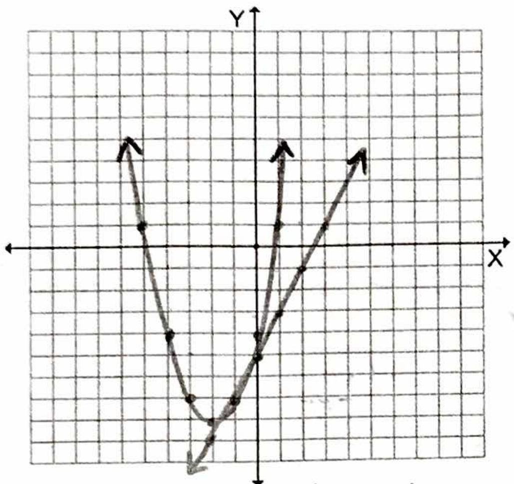
no solutions

9.6 Solving Nonlinear Systems of Equations

You try: $y = x^2 + 4x - 4$

$y = 2x - 5$

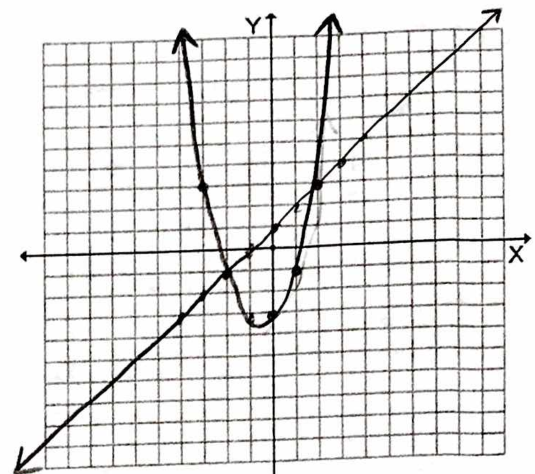
$\frac{-4}{2} = -2$



Solution: $(-1, -7)$

You try: $y = x^2 + x - 3$

$y = x + 1$



Solutions: $(2, 3)$

$(-2, -1)$

I can solve systems of nonlinear equations algebraically

Example 2: Solving a nonlinear system by substitution

Solve each of the following systems by substitution.

a) $y = -5x$

$y = x^2 - 3x - 3$

$-5x = x^2 - 3x - 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\boxed{x = -3}$ $\boxed{x = 1}$

b) $y = -2x^2$

$y = 3x + 2$

$-2x^2 = 3x + 2$

$0 = 2x^2 + 3x + 2$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{9 - 16}}{4}$

$x = \frac{-3 \pm \sqrt{-7}}{4}$

$\boxed{\text{no solution}}$

You try: $y = x^2 + 9$

$y = 9$

$9 = x^2 + 9$

$0 = x^2$

$\boxed{x = 0}$

You try: $y = x^2 + 4x$

$y = -4$

$-4 = x^2 + 4x$

$0 = x^2 + 4x + 4$

$0 = (x+2)^2$

$0 = x + 2$

$\boxed{x = -2}$

9.6 Solving Nonlinear Systems of Equations

Example 3: Solving a nonlinear system by elimination
Solve each of the following systems by elimination.

a) $y = x^2 + x$
$y = x + 5$

$x^2 + x = x + 5$
$x^2 = 5$
$x = \sqrt{5}$
$\boxed{x = \pm 2.24}$

b) $y = 2x + 5$
$y = -3x^2 + x - 4$

$2x + 5 = -3x^2 + x - 4$
$0 = -3x^2 - x - 9$
$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-3)(-9)}}{2(-3)}$
$x = \frac{1 \pm \sqrt{1 - 108}}{-6}$
$x = \frac{1 \pm \sqrt{107}}{-6}$ $\boxed{\text{no solution}}$

You try: $y = x^2$
$y = x - 3$

$x^2 = x - 3$
$0 = x^2 - x + 3$
$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(3)}}{2}$
$x = \frac{1 \pm \sqrt{1 - 12}}{2}$
$x = \frac{1 \pm \sqrt{-11}}{2}$ $\boxed{\text{no solution}}$

You try: $y = x^2 + 2x - 5$
$y = 2x - 1$

$2x - 1 = x^2 + 2x - 5$
$0 = x^2 - 4$
$4 = x^2$
$\boxed{x = \pm 2}$

I can approximate solutions of nonlinear systems and equations

Approximating Solutions: Sometimes you cannot find exact values for the solutions of a system of equations. It may be necessary to approximate the values of the solutions.

Example 4: Using your graphing calculator, approximate the solutions of the system and equation.

a) $y = x^2 - 2x - 5$
$y = 2^x + 1$

$\boxed{(-1.73, 1.31)}$

b) $-(3^x) + 3 = x^2 - x$

$\boxed{x \approx -1.23 \quad x = 1}$

Closure: What I learned today was...