8.4 Graphing $f(x) = a(x - h)^2 + k$

Key

Learning Target:

- How can you describe the graph of $f(x) = a(x - h)^2$?

Success Criteria:

- I can graph quadratic functions of the form $f(x) = a(x - h)^2$
- I can graph quadratic functions of the form $f(x) = a(x - h)^2 + k$
- I can identify even and odd functions
- I can model real-life problems using $f(x) = a(x - h)^2 + k$

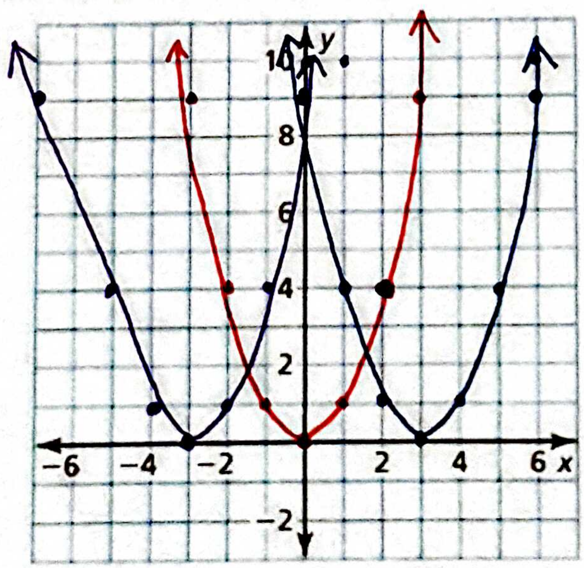
Exploration:

Work with a partner. Using different colors for each graph, graph the following equations on a single coordinate plane. You can use your graphing calculator to pick points.

$y = x^2$

$y = (x - 3)^2$

$y = (x + 3)^2$



What is similar about the graphs?

all open up; shape is the same

What is different about the graphs?

$y = (x-3)^2$ is shifted to the right 3 units; $y = (x+3)^2$ left 3 units

What do you think caused these differences?

the −3 and +3 in parentheses (opposite direction of the sign)

8.4 Graphing $f(x) = a(x - h)^2 + k$

I can graph quadratic functions of the form $f(x) = a(x - h)^2$

Graphing $f(x) = a(x - h)^2$

- When $h>0$, the graph of $f(x)=a(x-h)^2$ is a horizontal translation h units right of $f(x)=ax^2$.

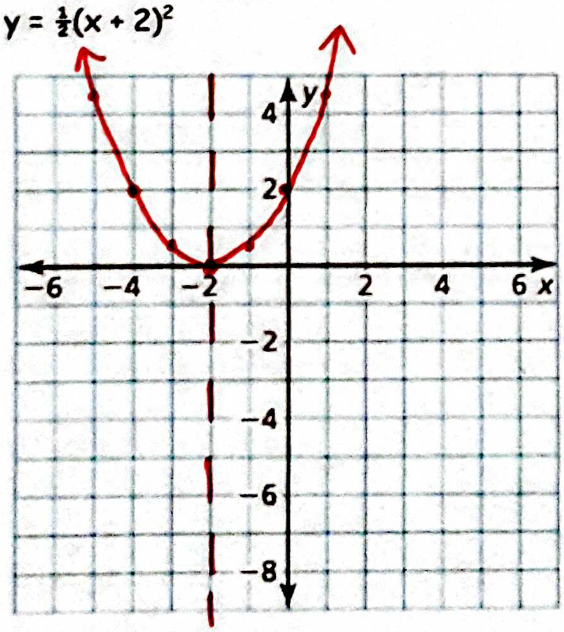
- When $h<0$, the graph of $f(x)=a(x-h)^2$ is a horizontal translation h units left of $f(x)=ax^2$.

Format: $f(x)=a(x+h)^2$ since $x--\#$
$x+\#$

Note: The vertex of the graph of $f(x) = a(x - h)^2$ is $(h,0)$, and the axis of symmetry is $x=h$.

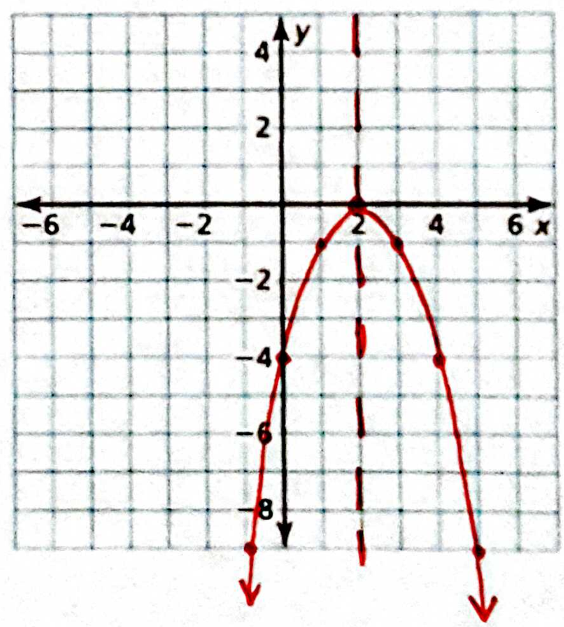
Example 1: Graph the following. Compare each graph to the graph of $f(x) = x^2$.

a) $y = \frac{1}{2}(x + 2)^2$



Comparison: wider; left 2
vertex @ $(-2,0)$

You try: b) $y = -(x - 2)^2$



Comparison: reflected over x-axis
right 2 units

8.4 Graphing $f(x) = a(x - h)^2 + k$

I can graph quadratic functions of the form $f(x) = a(x - h)^2 + k$

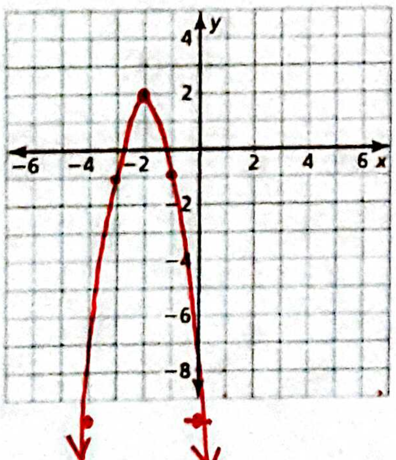
Graphing $f(x) = a(x - h)^2 + k$

Vertex form: $f(x)=a(x-h)^2+k$ where $a \neq 0$. The graph is a translation h units horizontally and k units vertically of the graph $f(x)=ax^2$.

Note: The vertex of the graph is (h,k) and the axis of symmetry is $x=h$.

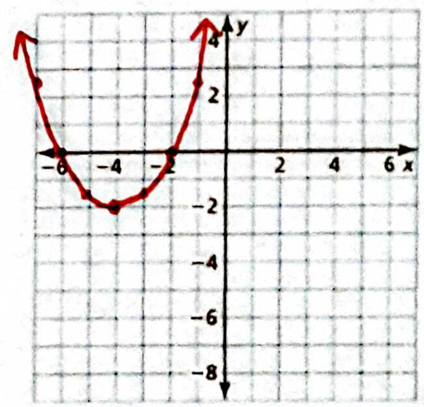
Example 2: Graph each of the following. Compare each graph to the graph of $f(x) = x^2$.

a) $g(x) = -3(x + 2)^2 + 2$



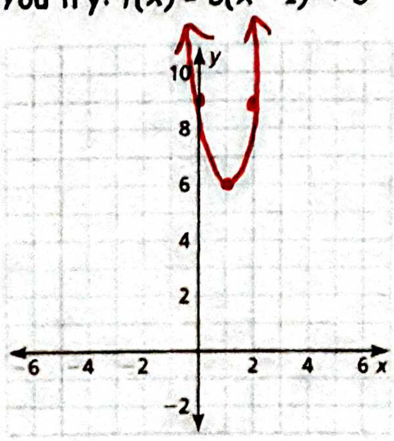
Comparison: vertex @ $(-2,2)$
reflection; vertical stretch

b) $h(x) = \frac{1}{2}(x + 4)^2 - 2$



Comparison: vertex @ $(-4,-2)$
horizontal stretch (vertical compression)

You try: $f(x) = 3(x - 1)^2 + 6$



Comparison: vertex @ $(1,6)$
vertical stretch

8.4 Graphing $f(x) = a(x - h)^2 + k$

I can identify even and odd functions

Even and Odd Functions

- A function $y = f(x)$ is even when $f(-x) = f(x)$ for each x in the domain of f. The graph is symmetric about the y-axis.
- A function $y = f(x)$ is odd when $f(-x) = -f(x)$ for each x in the domain of f. The graph is symmetric about the origin (it looks the same after reflections in the x-axis and then in the y-axis.

Example 3: Determine whether each function is *even, odd,* or *neither.*

a) $f(x) = 3x$

$f(-x) = 3(-x)$
$= -3x$
$= -f(x)$

$\boxed{\text{odd}}$

b) $g(x) = 2^x$

$g(-x) = 2^{-x}$
$= \left(\frac{1}{2}\right)^x$

$\boxed{\text{neither}}$

c) $h(x) = 3x^2 - 2x + 4$

$h(-x) = 3(-x)^2 - 2(-x) + 4$
$= 3x^2 + 2x + 4$

$h(x) \neq h(-x)$
$h(-x) \neq -h(x)$

$\boxed{\text{neither}}$

You try:

a) $f(x) = 5x$

$f(-x) = 5(-x)$
$= -5x$
$= -f(x)$

$\boxed{\text{odd}}$

b) $g(x) = 2x^2 - 6$

$g(-x) = 2(-x)^2 - 6$
$= 2x^2 - 6$
$= g(x)$

$\boxed{\text{even}}$

c) $h(x) = 2x^2 + 3$

$h(-x) = 2(-x)^2 + 3$
$= 2x^2 + 3$
$= h(x)$

$\boxed{\text{even}}$

Closure: What I learned today was ...

8.5 Using Intercept Form

Key

Learning Target:

- What are some of the characteristics of the graph of $f(x) = a(x-p)(x-q)$?

Success Criteria:

- I can graph quadratic functions of the form $f(x) = a(x-p)(x-q)$
- I can use intercept form to find zeros of functions
- I can use characteristics to graph and write quadratic functions

I can graph quadratic functions of the form f(x) = a(x – p)(x –q)

Graphing $f(x) = a(x-p)(x-q)$

- The x- intercepts of the graph are p and q
- Recall: the vertex is the halfway point between the zeros, so the axis of symmetry is $x = \frac{p+q}{2}$

Example 1: Graph the quadratic function $f(x) = -(x+3)(x-1)$

Step 1: Find *x*-intercepts (-3, 0)

(1 , 0)

Step 2: Find the vertex

AOS:

$x = \frac{-3+1}{2} = -1$

$f(-1) = -(-1+3)(-1-1)$

$= -(2)(-2)$

$= 4$

$\boxed{(-1, 4)}$

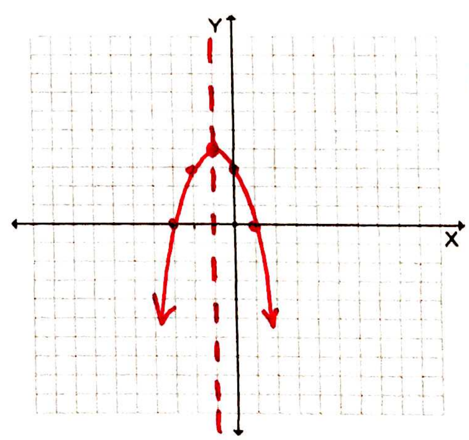
Step 3: Find *y*-intercept (0 , 3)

$f(0) = -(0+3)(0-1)$

$= -(3)(-1)$

$= 3$

Step 4: Draw the graph



8.5 Using Intercept Form

You try: Graph the quadratic function $f(x) = 4(x+1)(x+3)$

Step 1: Find x-intercepts $(-1, 0)$

$(-3, 0)$

Step 2: Find the vertex

AOS:

$x = \frac{-3+(-1)}{2} = -2$

$f(-2) = 4(-2+1)(-2+3)$

$= 4(-1)(1)$

$= -4$

$\boxed{(-2,-4)}$

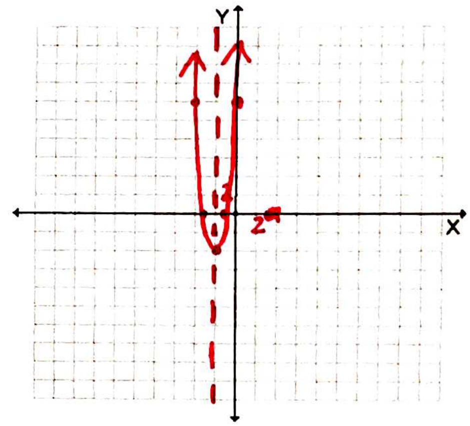
Step 3: Find y-intercept $(0, 12)$

$f(0) = 4(0+1)(0+3)$

$= 4(3)$

$= 12$

Step 4: Draw the graph



Example 2: Use the function $f(x) = -4x^2 + 36$

a) Graph $f(x)$

$f(x) = -4(x^2-9)$

$= -4(x+3)(x-3)$

x-ints

$(-3,0)$

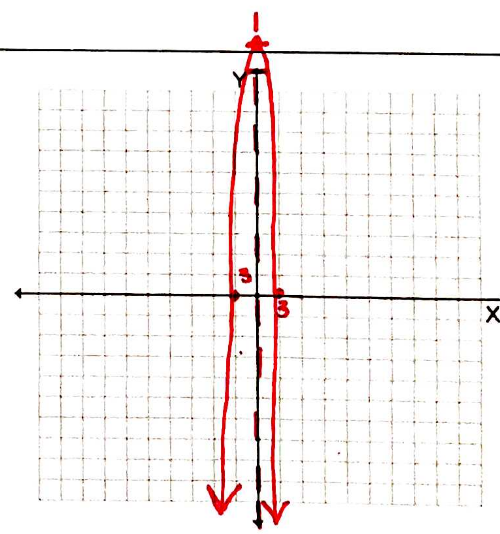
$(3,0)$

y-int

$(0,36)$

vertex: $\frac{-3+3}{2} = 0$

$\boxed{(0,36)}$



b) Describe the domain and range of $f(x)$.

Domain: $\mathbb{R}$

Range: $y \leq 36$

8.5 Using Intercept Form

You try: Use the function $f(x) = 2x^2 - 8$

a) Graph $f(x)$

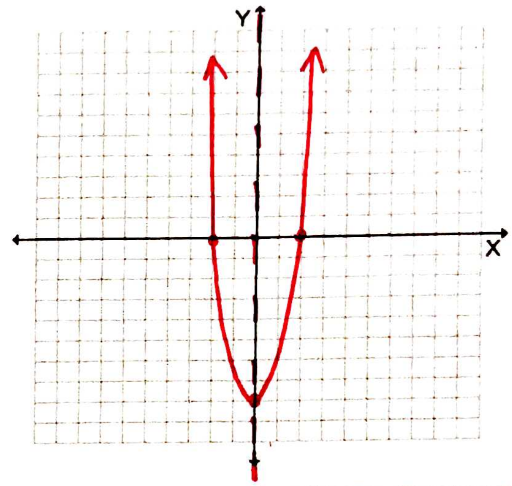
$f(x) = 2(x^2 - 4)$
$= 2(x+2)(x-2)$

vertex: $\frac{-2+2}{2} = 0$

$\boxed{(0,-8)}$

x-ints
(2,0)
(-2,0)

y-int
(0,-8)



b) Describe the domain and range of $f(x)$.

Domain: $\mathbb{R}$

Range: $y \geq -8$

I can use intercept form to find zeros of functions

Recall: The zeros of a function are the X - intercepts of the graph.

Example 3: Find the **zeros** of each function.

a) $y = -5x(x-2)$

$-5x = 0$ $\quad$ $x - 2 = 0$

$\boxed{x = 0}$ $\quad$ $\boxed{x = 2}$

b) $y = 3x^2 + x - 2$

$y = (3x^2 + 3x)(-2x - 2)$

$y = 3x(x+1) - 2(x+1)$

$y = (3x-2)(x+1)$

$3x - 2 = 0$ $\quad$ $x + 1 = 0$

$\boxed{x = \frac{2}{3}}$ $\quad$ $\boxed{x = -1}$

-6
3 -2
1

You try: $y = x(x^2 - 1)$

$y = x(x+1)(x-1)$

$\boxed{x = 0}$ $\quad$ $x + 1 = 0$ $\quad$ $x - 1 = 0$

$\boxed{x = -1}$ $\quad$ $\boxed{x = 1}$

8.5 Using Intercept Form

Example 4: Use zeros to graph the function $g(x) = -3x^2 - 6x + 24$

$g(x) = -3(x^2 + 2x - 8)$

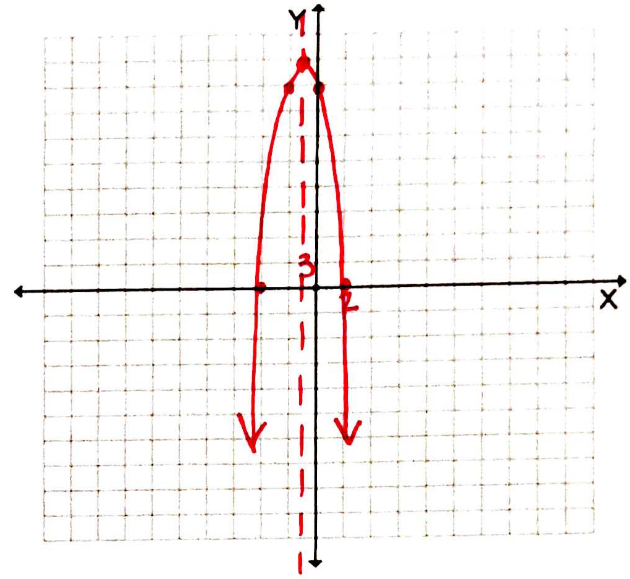
$g(x) = -3(x+4)(x-2)$

zeros: $x = -4$, $x = 2$

y-int: $(0, 24)$

vertex: $\frac{-4+2}{2} = -1$ $\boxed{(-1, 27)}$

$g(-1) = -3(-1+4)(-1-2)$
$= -3(3)(-3) = 27$



You try: Use zeros to graph the function $h(x) = 5x^2 - 20x + 15$

$h(x) = 5(x^2 - 4x + 3)$

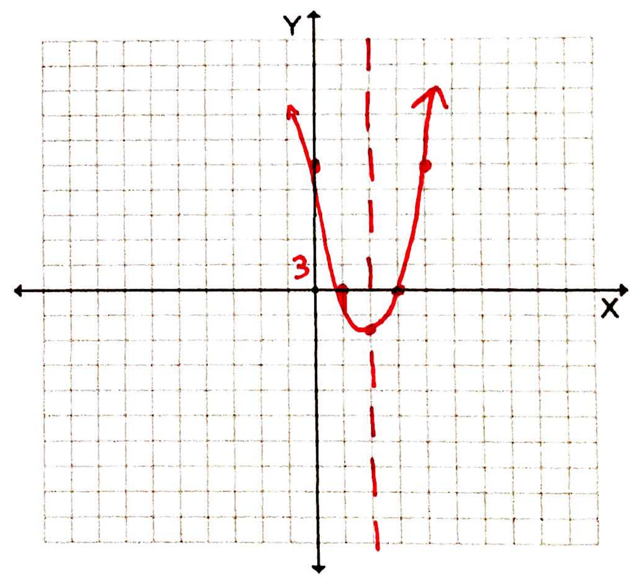
$h(x) = 5(x-3)(x-1)$

zeros: $x = 3$, $x = 1$

y-int: $(0, 15)$

vertex: $\frac{3+1}{2} = 2$ $(2, -5)$

$h(2) = 5(2-3)(2-1)$
$= 5(-1)(1)$
$= -5$



8.5 Using Intercept Form

I can use characteristics to graph and write quadratic functions

Recall: Other than standard form, two other forms of quadratics are:

Vertex form with equation $f(x)=a(x-h)^2+k$ and

Intercept form with equation $f(x)=a(x-p)(x-q)$

From these two equations, we can write the equation of a quadratic function that satisfies given conditions.

Example 5: Write a quadratic function in standard form whose graph satisfies the given condition(s):

a) A graph whose vertex is at $(\overset{h}{2}, \overset{k}{-5})$ and also passes through the point $(\overset{x}{4}, \overset{y}{7})$.

$y=a(x-h)^2+k$

$7=a(4-2)^2+(-5)$

$7=a(2)^2-5$

$12=4a$

$3=a$

$\boxed{y=3(x-2)^2-5}$

b) A graph whose vertex is at $(\overset{h}{-4}, \overset{k}{-3})$.

$\boxed{y=(x+4)^2-3}$

(since no other information is provided, the a value can vary)

You try: A graph whose vertex is at $(\overset{h}{-3}, \overset{k}{2})$ and also passes through the point $(\overset{x}{1}, \overset{y}{30})$

$y=a(x-h)^2+k$

$30=a(1--3)^2+2$

$30=a(4)^2+2$

$28=16a$

$\frac{7}{4}=a$

$\boxed{y=\frac{7}{4}(x+3)^2+2}$

8.5 Using Intercept Form

Example 6: Write a quadratic function in standard form whose graph satisfies the given condition(s).

a) A graph passing through the points $\underbrace{(\overset{p}{-9}, 0), (\overset{q}{-2}, 0)}_{\text{zeros}}$, and $(\overset{x}{-4}, \overset{y}{20})$

$$y = a(x-p)(x-q)$$

$$20 = a(-4+9)(-4+2)$$

$$20 = a(5)(-2)$$

$$20 = -10a$$

$$a = -2$$

$$\boxed{y = -2(x+9)(x+2)}$$

You try: A graph passing through the points $(\overset{p}{4}, 0)$, $(\overset{q}{-3}, 0)$, and $(\overset{x}{-2}, \overset{y}{50})$

$$y = a(x-p)(x-q)$$

$$50 = a(-2-4)(-2+3)$$

$$50 = a(-6)(1)$$

$$50 = -6a$$

$$a = -\frac{25}{3}$$

$$\boxed{y = \frac{-25}{3}(x-4)(x+3)}$$