8.3 Graphing $f(x) = ax^2 + bx + c$

Learning Target:

- How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

Success Criteria:

- I can graph quadratic functions of the form $f(x) = ax^2 + bx + c$.
- I can find the maximum and minimum values of quadratic functions.

I can graph quadratic functions of the form $f(x) = ax^2 + bx + c$.

Example 1: Use the quadratic function $f(x) = x^2 + 4x - 3$ for the following:

a) Find the values of a, b, and c and write them below.

$a = 1$ $\quad b = 4$ $\quad c = -3$

b) Will the parabola graph open upward ∪ or open downward ∩? Why? $a > 0$

c) Will the function have a minimum? ✓ opens up

Will it have a ~~maximum~~?

d) Find the axis of symmetry: Draw it as a dotted line on the graph below: $x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2$

$x = -2$

e) Find the x-coordinate of the vertex:

$x = -2$

f) Find the y-coordinate of the vertex:

$y = x^2 + 4x - 3 \quad = (-2)^2 + 4(-2) - 3$

$4 + (-8) - 3$

$y = -7$

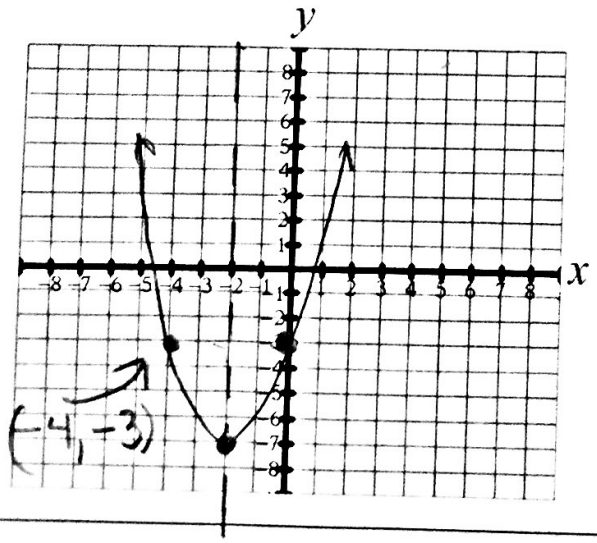
g) Write the vertex as an ordered pair:

Vertex is $(-2, -7)$

h) Find the y-intercept: (Let $x = 0$)

y-intercept is $(0, -3)$

i) Graph the function using the vertex, the y-intercept, and a third point using symmetry.



j) What is the minimum value of the function?

It's the y-coordinate of the vertex

$y = -7$

8.3 Graphing $f(x) = ax^2 + bx + c$

You try: Use the quadratic function $f(x) = -\frac{1}{2}x^2 - 3x + 2$ for the following:

a) Pick out the values of a, b, and c and write them below.

$a = -\frac{1}{2}$ $b = -3$ $c = 2$

b) Will the parabola graph open upward ∪ or open downward ∩? Why? $a < 0$

c) Will the function have a ~~minimum~~?

Will it have a maximum? ∩ opens down

d) Find the axis of symmetry:

$$x = \frac{-b}{2a} = \frac{-(-3)}{2(-\frac{1}{2})} = -3$$ $x = -3$

(*draw it as a dotted line on the graph below)

e) Find the x-coordinate of the vertex:

$x = -3$

f) Find the y-coordinate of the vertex:

$$y = -\frac{1}{2}x^2 - 3x + 2$$

$$= -\frac{1}{2}(-3)^2 - 3(-3) + 2$$

$$= -\frac{9}{2} + 9 + 2 = 11 - \frac{9}{2} = \frac{22}{2} - \frac{9}{2}$$

$y = \frac{13}{2}$ or 6.5

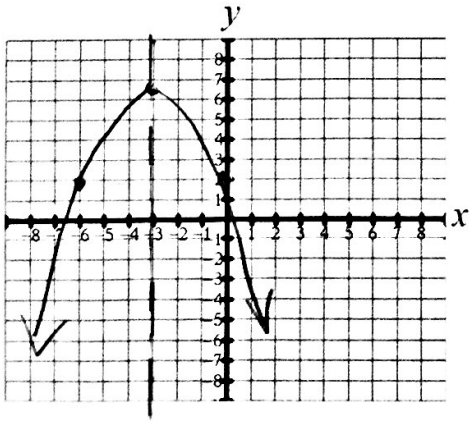
g) Write the vertex as an ordered pair:

Vertex is $(-3, \frac{13}{2})$ or $(-3, 6.5)$

h) Find the y-intercept: Let $x = 0$

y-intercept is $(0, 2)$

i) Graph the function using the vertex, the y-intercept, and a third point using symmetry.



j) What is the maximum value of the function?

It's the y-coordinate of the vertex:

$y = 6.5$

8.3 Graphing $f(x) = ax^2 + bx + c$

**Note: As we did in our section 8.2 lesson, we could have made a table of several ordered pairs for $f(x) = x^2 + 8x - 5$ and then plotted the points to graph the function.

However, the method we used in our first two examples, which is to use the vertex, the y-intercept, ar a third point using symmetry, is a quicker method.

I can find the maximum and minimum values of quadratic functions.

To find the vertex of a parabola using a graphing calculator:

While on the graph screen,

STEP #1: Press [2ND] then press [TRACE] then choose minimum OR maximum

STEP #2: Move the cursor to set a Left Bound then press [ENTER],
then set a Right Bound then press [ENTER],
and then press [ENTER] again.

Use a graphing calculator to find the vertex of the parabola graph for the given quadratic function, th give the maximum and the minimum value of the function.

Example: $y = \frac{2}{3}x^2 - 4x + 2$

Vertex: (3, -4)

Minimum Value: $y = -4$

Maximum Value: none

You try: $f(x) = -2x^2 - 5x + 15$

Vertex: (-1.25, 18.125)*

Minimum Value: none

Maximum Value: $y = 18.125$

* Press [WINDOW] [Ymax] 20 [ENTER] to get the vertex in the viewing window.

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Solving a Real-Life Problem: Suppose you have 200 feet of fencing and you are going to use it to enclose a rectangular space for your vegetable garden. You obviously want to have the largest area possible for the garden. What dimensions should you use?

a) Complete the table to find several different possible combinations for the Length L and the Width W for the rectangular garden. **(You may use a calculator)**
Note: The perimeter is 200 feet, so the length and width must total half of that (or 100 feet).

Width W (in feet)	Length L (in feet)	Area $A = L \cdot W$ (in square feet)
10	90	900
20	80	1600
33	67	2211
45	55	2475
62	38	2356
70	30	2100
85	15	1275
x	$100-x$	$x(100-x)$

b) Use the last row of your table to write an equation for the Area A when the width is x.

$A = x(100-x) = 100x - x^2 \Rightarrow A = -x^2 + 100x$

c) Graph this Area equation on your calculator.
Note: You will need to change the WINDOW values to see the vertex. Use the values in the table to get a rough idea of what your window settings need to be. Xmax = 100 Ymax = 2500
Xscl = 10 Yscl = 100

d) Use your graph to find the maximum possible area. Record this value below, along with the width and length needed to produce this maximum area (give the correct units as well).

Maximum Area = 2500 Width = x = 50 Length = $100 - 50 = 50$

Closure: what I learned today was...