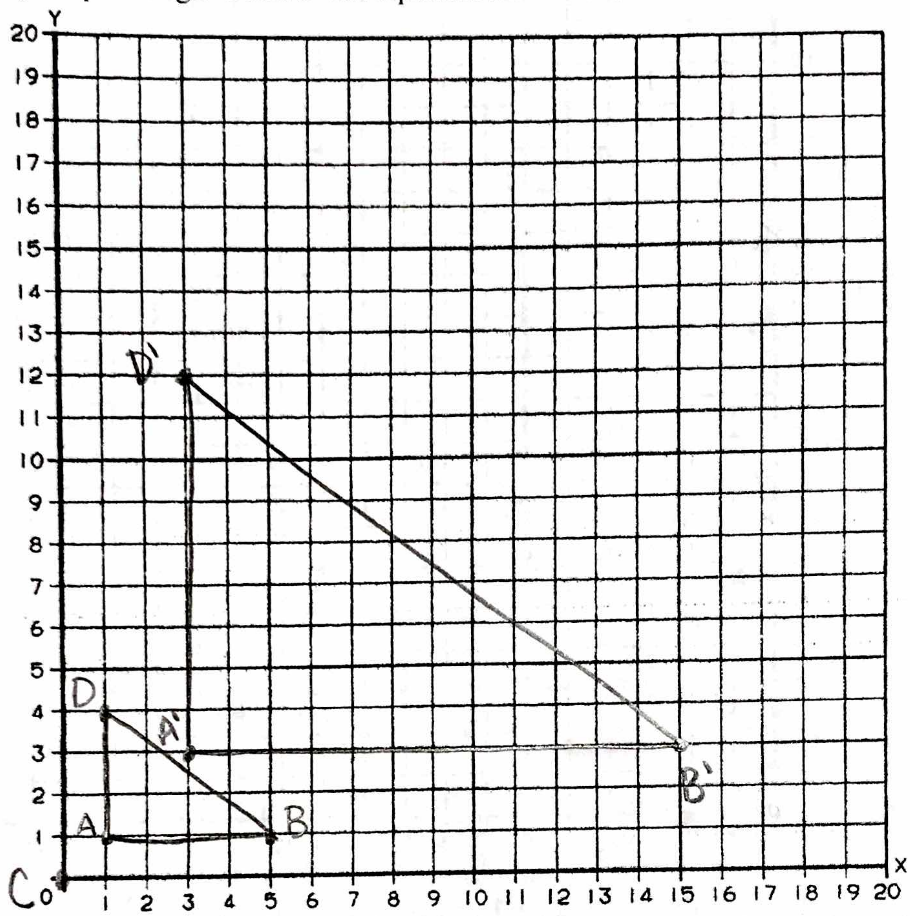
GEOMETRY NOTES
LESSON 31: Dilations

EXPLORATION: Dilations

1. Graph the following points on the coordinate plane: C (0, 0); A(1, 1); B(5, 1); D(1, 4). Connect points A, B, and D. This triangle will represent your pre-image. Point C will represent the center of dilation.



a. Using a different color, graph a new triangle on the coordinate plane using the following points: A'(3, 3), B' (15, 3), and D'(3, 12).

b. Complete the calculations listed below.

Pre-Image Length	Image Length	$\frac{Image}{Pre-Image}$
AB= 4	A'B'= 12	$\frac{12}{4}$
AD= 3	A'D'= 9	$\frac{9}{3}$
BD= 5	B'D'= 15	$\frac{15}{5}$

Simplifies to $\frac{3}{1}$

c. How do the image measurements compare to the pre-image measurements?

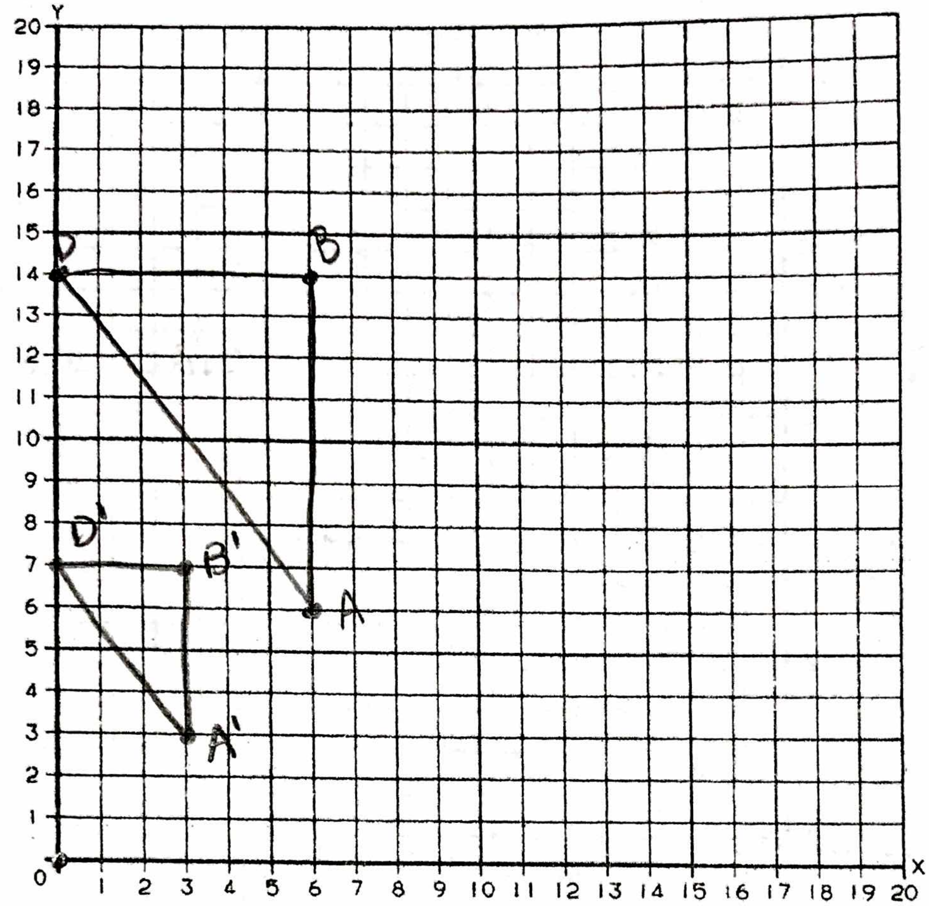
The image is 3 times the length of the pre-image

d. There are two types of dilations **enlargements** and **reductions**. What type of dilation is this?

Enlargement

GEOMETRY NOTES
LESSON 31: Dilations

2. Graph the following points on the coordinate plane: C (0, 0); A(6, 6); B(6, 14); D(0,14). Connect points A, B, and D. This triangle will represent your pre-image. Point C will represent the center of dilation.



a. Using a different color, graph a new triangle on the coordinate plane using the following points: A'(3, 3), B' (3, 7), and D'(0, 7).

b. Complete the calculations listed below.

Pre-Image Length	Image Length	$\frac{\textit{Image}}{\textit{Pre - Image}}$
AB= 8	A'B'= 4	$\frac{4}{8}$
AD= 10	A'D'= 5	$\frac{5}{10}$
BD= 6	B'D'= 3	$\frac{3}{6}$

simplifies to $\frac{1}{2}$

c. How do the image measurements compare to the pre-image measurements?

The image is half the length of the pre-image

d. There are two types of dilations **enlargements** and **reductions**. What type of dilation is this?

Reduction

3. What type of dilation would have a scale factor of $\frac{1}{5}$?

Reduction

4. What type of dilation would have a scale factor of $\frac{4}{1}$?

Enlargement

5. Looking back, what is the relationship between the scale factor and the type of dilation?

Enlargement → sf > 1   Reduction → sf < 1

6. Predict what would happen if you applied a scale factor of 1 to a set of coordinates.

The image would be the same as the pre-image

7. If you are the given the coordinates of the pre-image and the scale factor, how could you find the coordinates of the image without graphing?
(HINT: Look back at the coordinates of A, B, and D and the coordinates of A', B', and D' for both graphs.)

Multiply the pre-image coordinates by the scale factor

Summary

DILATIONS

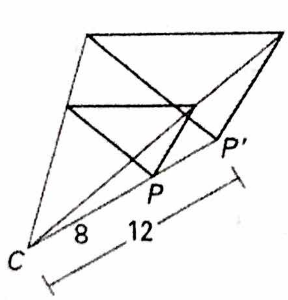
Dilation: Transformation that creates similar figures by increasing or decreasing in size

Scale Factor = the amount a preimage increases by to get the image

- **Enlargement:** gets larger **(Scale factor is** K>1**)**
- **Reduction:** gets smaller **(Scale factor is** K<1**)**

EXAMPLES: Identify the type of dilation and find the scale factor.

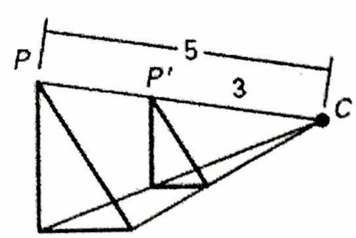
1.



Type of Dilation: Enlargement

Scale Factor: $\frac{12}{8} = \boxed{\frac{3}{2}}$

2.

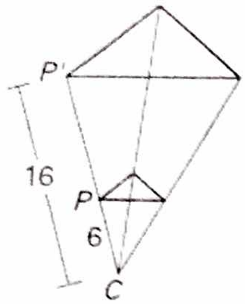


Type of Dilation: Reduction

Scale Factor: $\boxed{\frac{3}{5}}$

YOU TRY:

3.

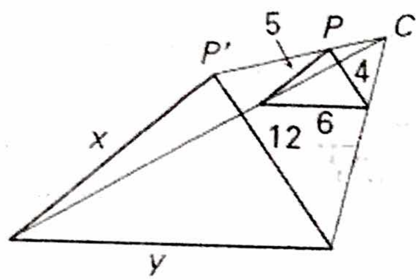


Type of Dilation: Enlargement

Scale Factor: $\frac{16}{6} = \boxed{\frac{8}{3}}$

EXAMPLES: Identify the dilation and scale factor. Then find the values of the variables

4.



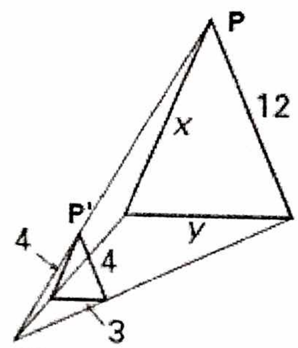
Type of Dilation: Enlargement

Scale Factor: $\frac{12}{4} = \boxed{3}$

x= 15

y= 18

5.



Type of Dilation: Reduction

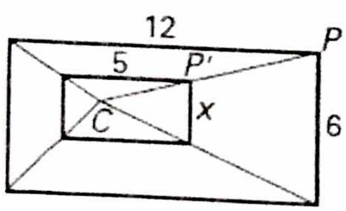
Scale Factor: $\frac{4}{12} = \boxed{\frac{1}{3}}$

x= 12

y= 9

YOU TRY:

6.



Type of Dilation: Reduction

Scale Factor: $\boxed{\frac{5}{12}}$

x= $\frac{30}{12} = \frac{15}{6} = \boxed{\frac{5}{2}}$

GEOMETRY NOTES
LESSON 32: Ratio and Proportion and Problem Solving with Proportions

Ratio: comparison of two quantities w/ same units

Proportion: equation with two ratios

If $\frac{a}{b} = \frac{c}{d}$ Then $ad = bc$

cross multiply

EXAMPLES: Solving the proportions.

1. $\frac{9}{15} = \frac{6}{x}$

$9x = 6 \cdot 15$

$9x = 90$

$\boxed{x = 10}$

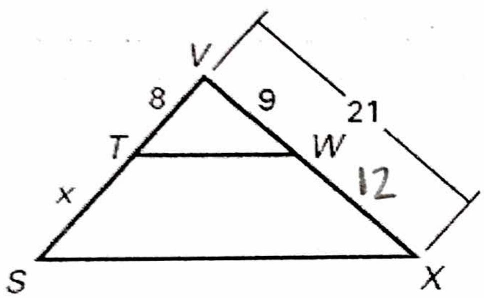
2. $\frac{4}{x-4} = \frac{3}{x}$

$4x = 3(x-4)$

$4x = 3x - 12$

$\boxed{x = -12}$

3. Given: $\frac{ST}{TV} = \frac{WX}{VW}$. Find ST.



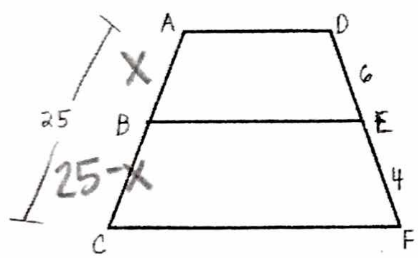
$\frac{x}{8} = \frac{12}{9}$

$9x = 96$

$x = 10.\overline{6}$

$\boxed{ST = 10.\overline{6}}$

4. Given: $\frac{AB}{BC} = \frac{DE}{EF}$. Find BC.



$\frac{x}{25-x} = \frac{6}{4}$

$4x = 6(25-x)$

$4x = 150 - 6x$

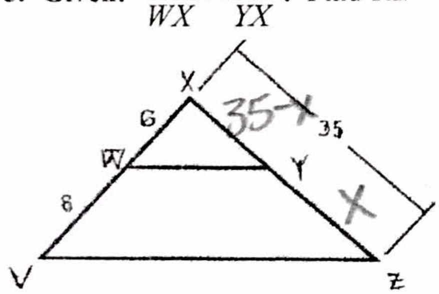
$10x = 150$

$x = 15$

$BC = 25 - 15$

$\boxed{BC = 10}$

5. Given: $\frac{VW}{WX} = \frac{ZY}{YX}$. Find YZ.



$\frac{8}{6} = \frac{x}{35-x}$

$6x = 8(35-x)$

$6x = 280 - 8x$

$14x = 280$

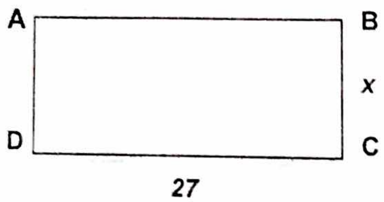
$x = 20$

$\boxed{YZ = 20}$

GEOMETRY NOTES
LESSON 32: Ratio and Proportion and Problem Solving with Proportions

EXAMPLES: Solve.

6. The ratio of BC: DC is 2:9. Find x.

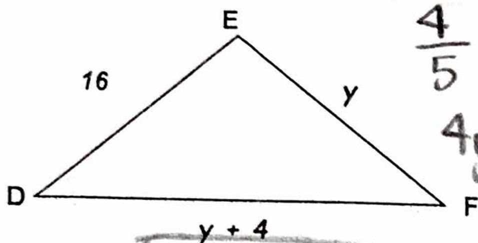


$\frac{2}{9} = \frac{x}{27}$

$9x = 54$

$\boxed{x = 6}$

7. The ratio DE: EF: DF= 4:5:6. Find EF and DF.



$\frac{4}{5} = \frac{16}{y}$

$4y = 80$

$y = 20$

OR

$\frac{5}{6} = \frac{y}{y+4}$

$6y = 5(y+4)$

$6y = 5y + 20$

$y = 20$

OR

$\frac{4}{6} = \frac{16}{y+4}$

$4(y+4) = 96$

$4y + 16 = 96$

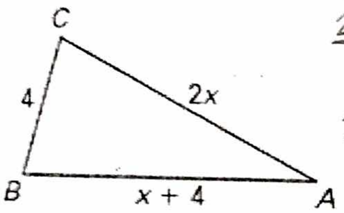
$4y = 80$

$y = 20$

$\boxed{EF = 20}$ $\boxed{DF = 24}$

YOU TRY:

8. The ratio of AC: BC: AB is 2:1:2. Find the length of each side.



$\frac{2}{1} = \frac{2x}{4}$

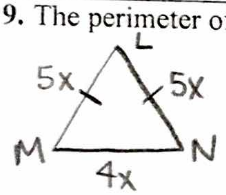
$2x = 8$

$x = 4$

$\boxed{\begin{array}{l} AC = 8 \\ BC = 4 \\ AB = 8 \end{array}}$

EXAMPLES: Using Ratios

9. The perimeter of isosceles Δ LMN is 56 in. The ratio of LM: MN is 5:4 and LM = LN. Find the lengths of the sides.



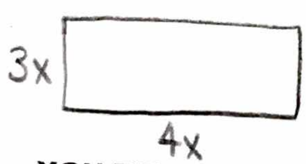
$5x + 5x + 4x = 56$

$14x = 56$

$x = 4$

$\boxed{\begin{array}{l} LM = 20\text{in} \\ LN = 20\text{in} \\ MN = 16\text{in} \end{array}}$

10. The area of a rectangle is 192 ft^2. The width:length ratio is 3:4. Find the width.



$3x \cdot 4x = 192$

$\frac{12x^2}{12} = \frac{192}{12}$

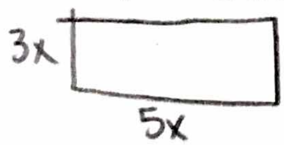
$x^2 = 16$

$x = 4$

$\boxed{\begin{array}{l} \text{Width} = 12\text{ft} \\ \text{length} = 16\text{ft} \end{array}}$

YOU TRY:

11. The perimeter of a rectangular room is 64 ft. The width:length ratio is 3:5. What are the dimensions of the room?



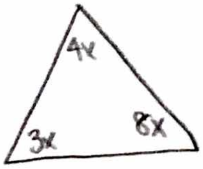
$3x + 5x + 3x + 5x = 64$

$16x = 64$

$x = 4$

$\boxed{\begin{array}{l} \text{Width} = 12\text{ ft} \\ \text{length} = 20\text{ft} \end{array}}$

12. The measures of the angles in a triangle are in the extended ratio of 3:4:8. Find the measures of the angles.



$4x + 3x + 8x = 180$

$15x = 180$

$x = 12$

$\boxed{36°, 48°, 96°}$

GEOMETRY NOTES

LESSON 33: Similar Polygons

EXPLORATION: Part 1

ΔABC and ΔDEF are similar. Let's explore what makes these polygons similar.

1. Cut out your triangles. Match up your angles. What do you notice about your angles? How do the corresponding angles compare?

COMPARE	CONGRUENT?
$\angle A$ with $\angle D$	______
$\angle B$ with $\angle E$	______
$\angle C$ with $\angle F$	______

2. Measure all the corresponding sides in both triangles. **MEASURE IN INCHES!** Then find the ratio of the corresponding sides. Convert it to a decimal. How do they compare? (Due to inaccurate measurements your ratios may not be accurate.)

BIG TRIANGLE	LITTLE TRIANGLE	RATIO OF SIDES (BIG:LITTLE)
AB______	DE______	______
BC______	EF______	______
AC______	DF______	______

3. Knowing that ΔABC and ΔDEF are similar and using what you found in your investigation, what must be true for polygons to be similar? Fill-in the blanks below. Explain how this differs from *congruent* polygons.

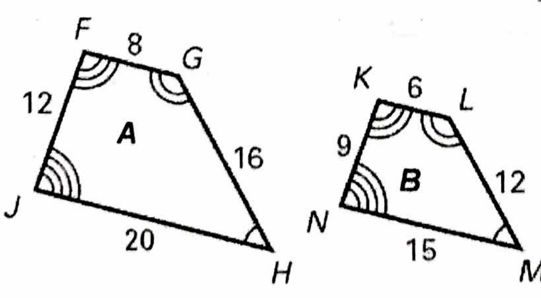
SIMILAR POLYGONS

A correspondence between two polygons such that:

1. Corresponding angles are congruent
2. Corresponding sides are proportional

EXAMPLES: Are these polygons similar? If so, find the scale factor of Figure A to Figure B.

1.



$\frac{8}{6} = \frac{4}{3}$

$\frac{12}{9} = \frac{4}{3}$

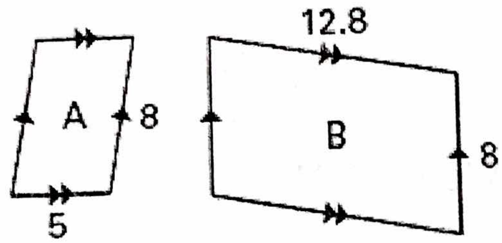
$\frac{20}{15} = \frac{4}{3}$

$\frac{16}{12} = \frac{4}{3}$

Similar

scale factor $= \frac{3}{4}$

2.



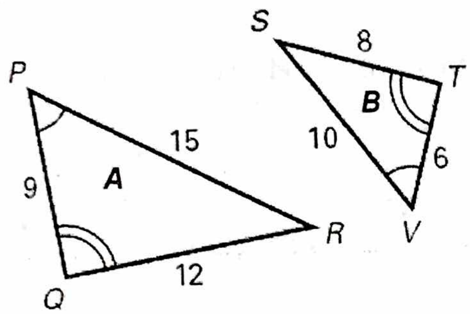
$\frac{12.8}{8} = \frac{8}{5}$

$\frac{8}{5}$

Similar
Scale factor $= \frac{5}{8}$

YOU TRY:

3.



$\frac{9}{6} = \frac{3}{2}$

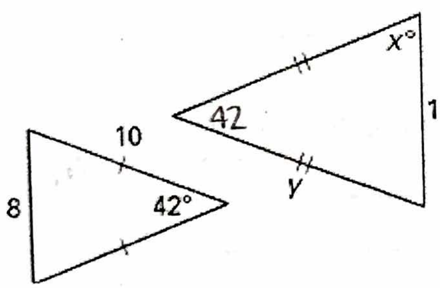
$\frac{12}{8} = \frac{3}{2}$

$\frac{15}{10} = \frac{3}{2}$

Similar
Scale factor $= \frac{3}{2}$

EXAMPLES: The two triangles are similar. Find the value of x and y.

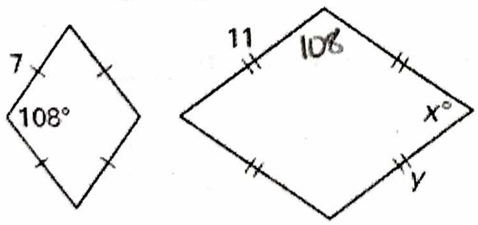
4.



$42 + x + x = 180$
$2x + 42 = 180$
$2x = 138$
$\boxed{x = 69}$

$\frac{10}{8} = \frac{y}{10}$
$8y = 100$
$\boxed{y = 12.5}$

5.

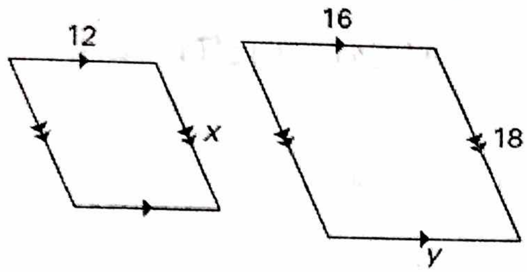


$x + 108 = 180$
$\boxed{x = 72}$
$\boxed{y = 11}$

GEOMETRY NOTES
LESSON 33: Similar Polygons

YOU TRY:

6.



$\frac{16}{12} = \frac{18}{x}$

$16x = 216$

$\boxed{x = 13.5}$

$\boxed{y = 16}$

EXPLORATION: Part 2

Find the perimeter and area of ΔABC and ΔDEF. Find the ratio of their perimeters and areas. Compare this to the ratio of the sides (scale factor) from the part 1.

PERIMETER OF BIG TRIANGLE	**PERIMETER OF LITTLE TRIANGLE**	**AREA OF BIG TRIANGLE**	**AREA OF LITTLE TRIANGLE**	**RATIO OF SIDES BIG:LITTLE**	**RATIO OF PERIMETERS BIG:LITTLE**	**RATIO OF AREAS BIG:LITTLE**

a. What do you notice about the relationship between the ratio of the sides (scale factor) and the ratio of the perimeters? equal

Ratio of sides: $\frac{a}{b}$ Perimeter: $\frac{a}{b}$

b. What do you notices about the relationship between the ratio of the sides (scale factor) and the ratio of the areas?

Scale factor squared is ratio of areas Area: $\left(\frac{a}{b}\right)^2$

EXAMPLES:

7. The ratio of the one side of ΔCDE to the corresponding side of similar triangle ΔFGH is 1:3. The perimeter of triangle ΔFGH is 36 and its area is 54. Find the perimeter and area of ΔCDE.

Perimeter: $\frac{1}{3} = \frac{x}{36}$

$3x = 36$

$x = 12$

$\boxed{\text{Perimeter } \Delta CDE = 12}$

Area: $\left(\frac{1}{3}\right)^2 = \frac{x}{54}$

$\frac{1}{9} = \frac{x}{54}$

$9x = 54$

$x = 6$ $\boxed{\text{Area } \Delta CDE = 6}$

8. Parallelogram PQRS is similar to parallelogram JKLM. The perimeter of parallelogram PQRS is 90 centimeters. The perimeter of parallelogram JKLM is 40 centimeters. The lengths of the sides of parallelogram PQRS are 18 cm and 27 cm. Find the length of the sides of parallelogram JKLM and the ratio of their areas.

$\frac{90}{40} = \frac{18}{x}$

$90x = 720$

$x = 8$

$\frac{90}{40} = \frac{27}{x}$

$90x = 1080$

$x = 12$

$\boxed{8\text{cm}, 12\text{ cm}}$

GEOMETRY NOTES

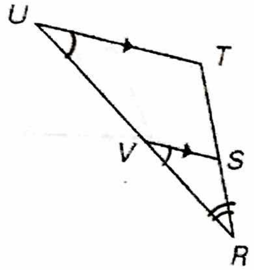
LESSON 34: Similar Triangles

SIMILARITY SHORTCUTS THAT WORK:

1. AA
2. SSS
3. SAS

EXAMPLES: Determine whether the triangles can be proved similar. If so, write a similarity statement.

1. Yes-AA $\triangle RSV \sim \triangle RTU$

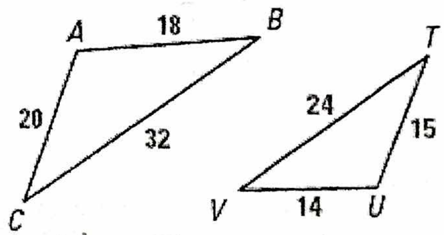


$\angle R \cong \angle R$

$\angle TUV \cong \angle SVR$

by alt int <s

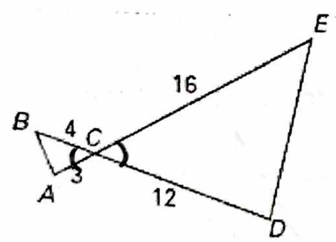
2. Not Similar



$\frac{14}{18} = \frac{7}{9}$ $\frac{15}{20} = \frac{3}{4}$

$\frac{24}{32} = \frac{3}{4}$

3. Yes - SAS $\triangle ABC \sim \triangle DEC$

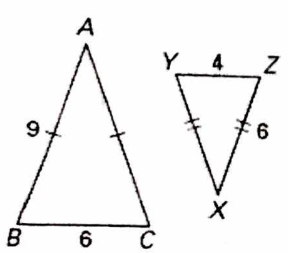


$\frac{16}{4} = \frac{4}{1}$

$\frac{12}{3} = \frac{4}{1}$

YOU TRY:

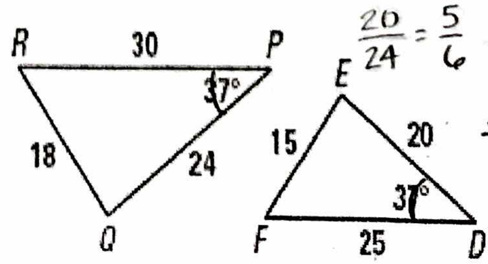
4. Yes-SSS $\triangle XYZ \sim \triangle ABC$



$\frac{4}{6} = \frac{2}{3}$

$\frac{6}{9} = \frac{2}{3}$

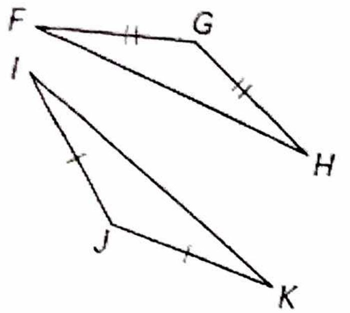
5. Yes-SAS $\triangle PQR \sim \triangle DEF$



$\frac{20}{24} = \frac{5}{6}$

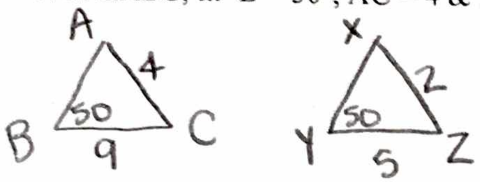
$\frac{25}{30} = \frac{5}{6}$

6. No - not enough info



EXAMPLES: **Determine if the triangles are similar. If so, state the reason why. Sketching the triangles will help.**

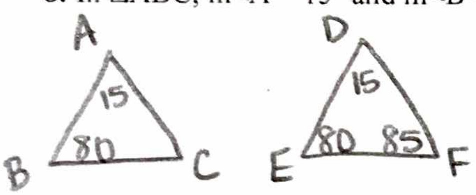
7. In ΔABC, m<B = 50°, AC = 4 & BC = 9. In ΔXYZ, m<Y = 50°, XZ = 2 & YZ = 5.



Not similar

YOU TRY:

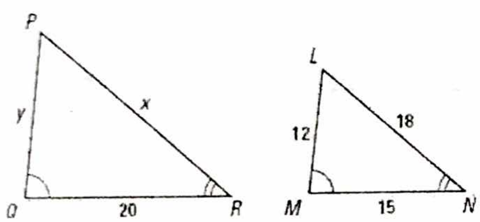
8. In ΔABC, m<A = 15° and m<B = 80°. In ΔDEF, m<E = 80° and m<F = 85°.



Similar - AA

EXAMPLES: **The triangles are similar. Find the values of the variables.**

9.



$\frac{15}{20} = \frac{18}{x}$

$15x = 360$

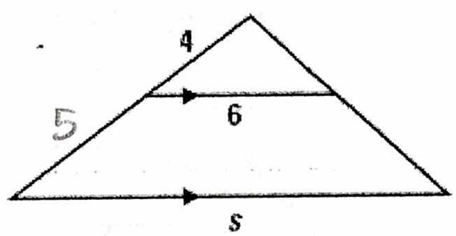
$\boxed{x = 24}$

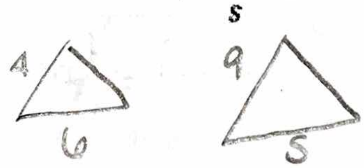
$\frac{15}{20} = \frac{12}{y}$

$15y = 240$

$\boxed{y = 16}$

10.





$\frac{4}{9} = \frac{6}{s}$

$4s = 54$

$\boxed{s = 13.5}$

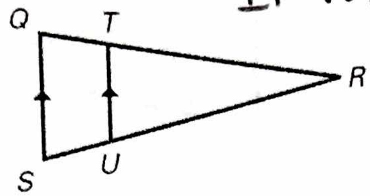
GEOMETRY NOTES

LESSON 35: Side Splitter Theorem

SIDE SPLITTER THEOREM

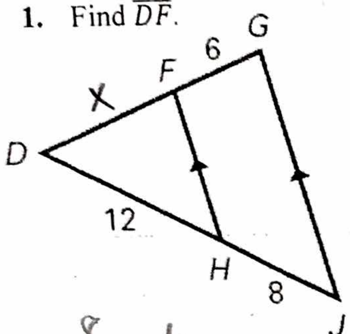
If a line parallel to one side of a triangle intersects the other two sides, then it divides sides proportionally.

If $\overline{TU} \parallel \overline{QS}$, then $\frac{\overline{RT}}{\overline{TQ}} = \frac{\overline{RU}}{\overline{US}}$



EXAMPLES: Find the length of the segment.

1. Find $\overline{DF}$.



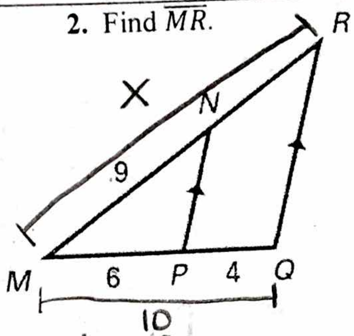
$\frac{8}{12} = \frac{6}{x}$

$8x = 72$

$x = 9$

$\boxed{DF = 9}$

2. Find $\overline{MR}$.



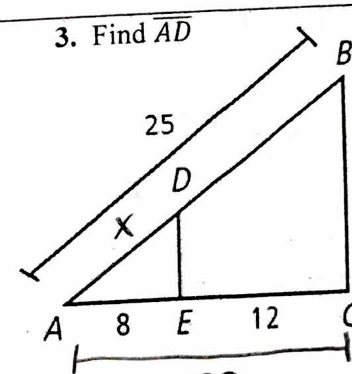
$\frac{6}{10} = \frac{9}{x}$

$6x = 90$

$x = 15$

$\boxed{MR = 15}$

3. Find $\overline{AD}$



$\frac{8}{20} = \frac{x}{25}$

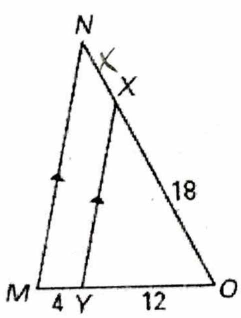
$20x = 200$

$x = 10$

$\boxed{AD = 10}$

YOU TRY:

4. Find $\overline{NX}$.



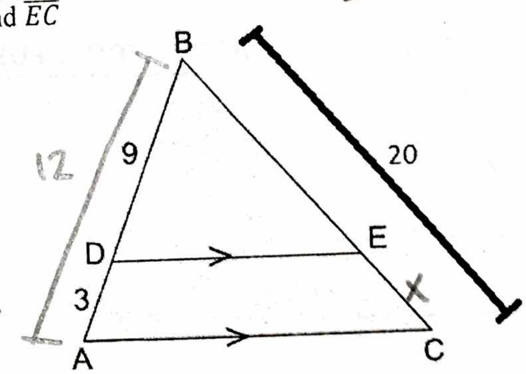
$\frac{4}{12} = \frac{x}{18}$

$12x = 72$

$x = 6$

$\boxed{NX = 6}$

5. Find $\overline{EC}$



$\frac{3}{12} = \frac{x}{20}$

$12x = 60$

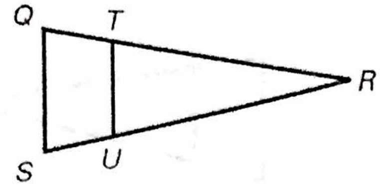
$x = 5$

$\boxed{EC = 5}$

GEOMETRY NOTES
LESSON 35: Side Splitter Theorem

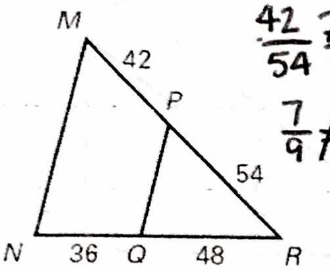
CONVERSE OF THE SIDE SPLITTER THEOREM

If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.



EXAMPLES: Determine whether the lines are parallel. You must show work (just a yes or no is not enough).

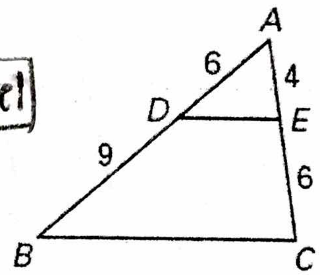
6. Is $\overline{MN}$ parallel to $\overline{PQ}$?



$\frac{42}{54} \stackrel{?}{=} \frac{36}{48}$

$\frac{7}{9} \neq \frac{3}{4}$ $\boxed{\text{Not parallel}}$

7. Is $\overline{BC}$ parallel to $\overline{DE}$?

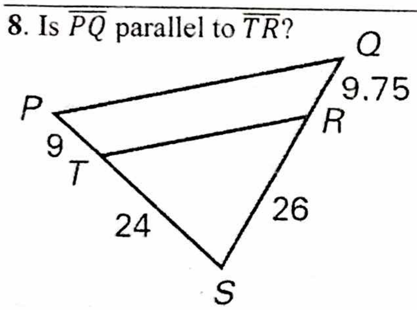


$\frac{6}{9} \stackrel{?}{=} \frac{4}{6}$

$\frac{2}{3} = \frac{2}{3}$ $\boxed{\text{Parallel}}$

YOU TRY:

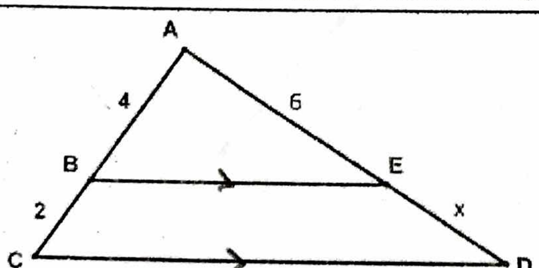
8. Is $\overline{PQ}$ parallel to $\overline{TR}$?



$\frac{9}{24} \stackrel{?}{=} \frac{9.75}{26}$

$\frac{3}{8} = \frac{3}{8}$ $\boxed{\text{Parallel}}$

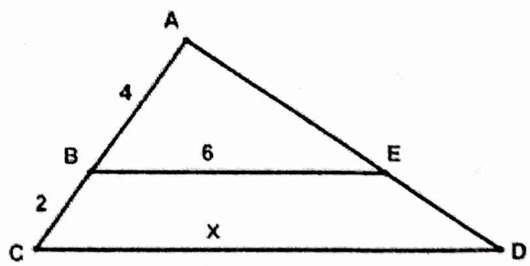
WHAT IS THE DIFFERENCE? SOLVE FOR X FOR EACH TRIANGLE. (BE // CD IN BOTH).

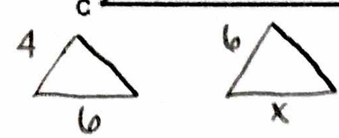


$\frac{4}{2} = \frac{6}{x}$

$4x = 12$

$\boxed{x = 3}$





$\frac{4}{6} = \frac{6}{x}$

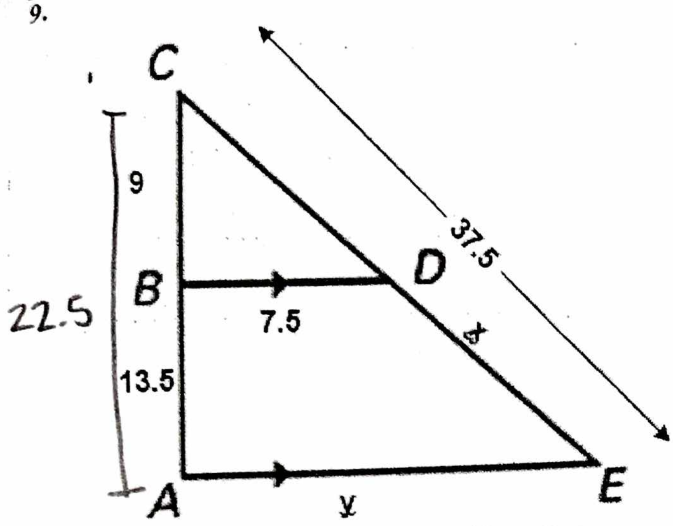
$4x = 36$

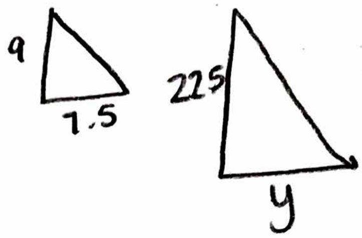
$\boxed{x = 9}$

LESSON 35: Side Splitter Theorem

EXAMPLES: Find the lengths of the missing segments.

9.





$$\frac{22.5}{37.5} = \frac{13.5}{x}$$

$$22.5x = 506.25$$

$$\boxed{x = 22.5}$$

$$\frac{9}{22.5} = \frac{7.5}{y}$$

$$9y = 168.75$$

$$\boxed{y = 18.75}$$