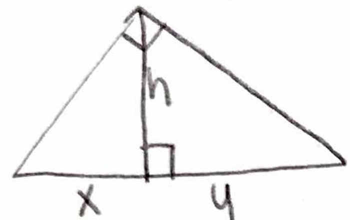
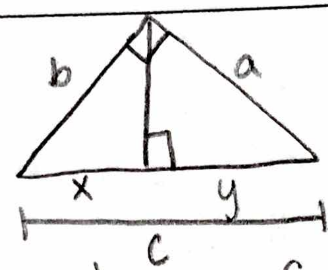
GEOMETRY NOTES

LESSON 36: Geometric Mean



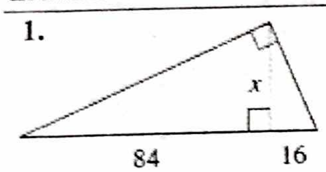
$$\frac{x}{h} = \frac{h}{y}$$



$$\frac{c}{b} = \frac{b}{x} \qquad \frac{c}{a} = \frac{a}{y}$$

EXAMPLES: Find the value of each variable.

1.



$$\frac{84}{x} = \frac{x}{16}$$

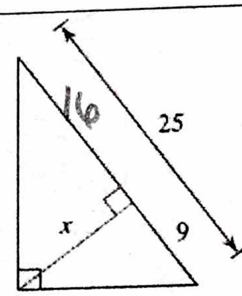
$$x^2 = 1344$$

$$x = \sqrt{1344}$$

$$x = \sqrt{21 \cdot 64}$$

$$\boxed{x = 8\sqrt{21}}$$

2.

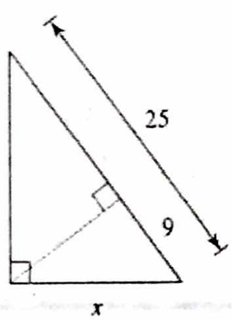


$$\frac{9}{x} = \frac{x}{16}$$

$$x^2 = 144$$

$$\boxed{x = 12}$$

3.

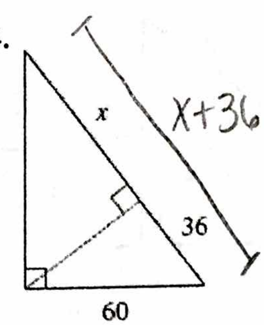


$$\frac{25}{x} = \frac{x}{9}$$

$$x^2 = 225$$

$$\boxed{x = 15}$$

4.



$$\frac{x+36}{60} = \frac{60}{36}$$

$$36(x+36) = 60^2$$

$$36x + 1296 = 3600$$

$$36x = 2304$$

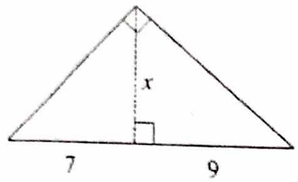
$$\boxed{x = 64}$$

GEOMETRY NOTES

LESSON 36: Geometric Mean

YOU TRY:

5.



$$\frac{7}{x}=\frac{x}{9}$$

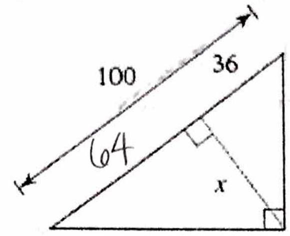
$$x^2=63$$

$$x=\sqrt{63}$$

$$x=\sqrt{9\cdot 7}$$

$$\boxed{x=3\sqrt{7}}$$

6.



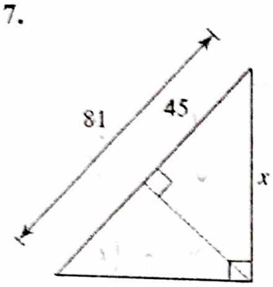
$$\frac{36}{x}=\frac{x}{64}$$

$$x^2=2304$$

$$x=\sqrt{2304}$$

$$\boxed{x=48}$$

7.



$$\frac{81}{x}=\frac{x}{45}$$

$$x^2=81\cdot 45$$

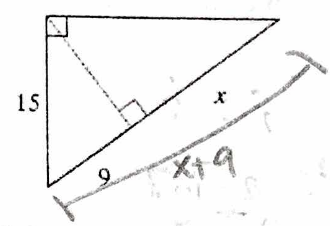
$$x=\sqrt{81\cdot 45}$$

$$x=\sqrt{81}\cdot\sqrt{45}$$

$$x=9\cdot\sqrt{9\cdot 5}$$

$$\boxed{x=27\sqrt{5}}$$

8.



$$\frac{x+9}{15}=\frac{15}{9}$$

$$9(x+9)=15^2$$

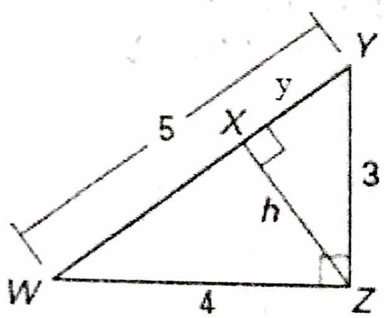
$$9x+81=225$$

$$9x=144$$

$$\boxed{x=16}$$

EXAMPLES: Find the value of each variable.

9.



$$\frac{5}{3}=\frac{3}{h}$$

$$5h=9$$

$$h=\frac{9}{5}$$

$$\boxed{h=1.8}$$

$$1.8^2+y^2=3^2$$

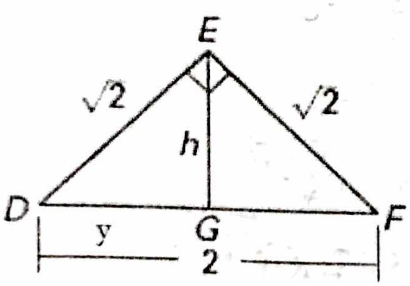
$$3.24+y^2=9$$

$$y^2=5.76$$

$$\boxed{y=2.4}$$

YOU TRY:

10.



$$\frac{2}{\sqrt{2}}=\frac{\sqrt{2}}{y}$$

$$2y=\sqrt{2}^2$$

$$2y=2$$

$$\boxed{y=1}$$

$$1^2+h^2=\sqrt{2}^2$$

$$1+h^2=2$$

$$h^2=1$$

$$\boxed{h=1}$$

GEOMETRY NOTES

LESSON 37: The Pythagorean Theorem and Its Converse

Pythagorean Theorem: Used with right triangles

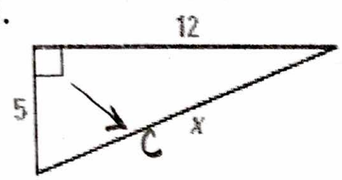
$a^2+b^2=c^2$ $c \rightarrow$ hypotenuse

Pythagorean Triple: Set of whole numbers that make $a^2+b^2=c^2$ true

Examples: 3,4,5 5,12,13 8,15,17 7,24,25 + any of their multiples

EXAMPLES: Find the unknown side length. Simplify answers that are radicals and tell whether the side lengths form a Pythagorean triple.

1.



5,12,13

Pythagorean triple

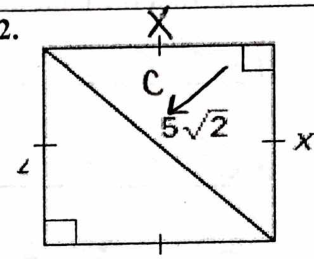
$5^2+12^2=x^2$

$25+144=x^2$

$169=x^2$

$\boxed{x=13}$

2.



$x^2+x^2=(5\sqrt{2})^2$

$2x^2=25\cdot 2$

$2x^2=50$

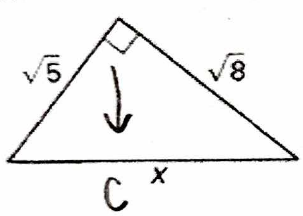
$x^2=25$

$\boxed{x=5}$

Not Pythagorean Triple

YOU TRY

3.



$\sqrt{5}^2+\sqrt{8}^2=x^2$

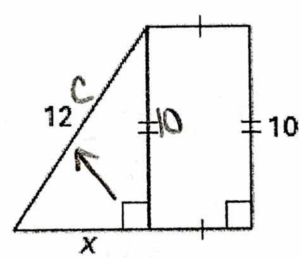
$5+8=x^2$

$13=x^2$

$\boxed{x=\sqrt{13}}$

Not Pythagorean Triple

4.



$10^2+x^2=12^2$

$100+x^2=144$

$x^2=44$

$x=\sqrt{44}$

$\boxed{x=2\sqrt{11}}$

Not Pythagorean Triple

GEOMETRY NOTES

LESSON 37: The Pythagorean Theorem and Its Converse

EXPLORATION:

1. Make an acute, obtuse, and right triangle with the Angleg sticks.
2. For each triangle, use the side lengths on the sticks to fill in the chart below.

Type of Triangle	Shortest side	Middle Side	Longest Side	$a^2 + b^2$ ____ c^2 (<, >, =)
Acute				
Right				
Obtuse				

Using the information you gathered from the table above, fill-in the if/then statements below.

If $c^2 = a^2 + b^2$, then right $\triangle$.

If $c^2 < a^2 + b^2$, then acute $\triangle$.

If $c^2 > a^2 + b^2$, then obtuse $\triangle$.

EXAMPLES: Decide whether the numbers can represent the side lengths of a triangle. If they can, classify each triangle as acute, right, or obtuse.

5. 8, 24, 18

c

$8+18>24$

$26>24$ ✓

$8^2+18^2 \stackrel{?}{=} 24^2$

$64+324 \stackrel{?}{=} 576$

$388 \neq 576$

$\boxed{\text{obtuse } \triangle}$

6. 2, 2, 4

c

$2+2>4$

$4>4$

$\boxed{\text{not a } \triangle}$

7. $14\sqrt{2}$, 14, 14

c

$14^2+14^2=(14\sqrt{2})^2$

$196+196=392$

$392=392$

$\boxed{\text{right } \triangle}$

YOU TRY:

8. 3.2, 4.8, 5.1

c

$3.2^2+4.8^2=5.1^2$

$10.24+23.04 \stackrel{?}{=} 26.01$

$33.28 \neq 26.01$

$\boxed{\text{acute } \triangle}$

GEOMETRY NOTES
LESSON 38: Special Right Triangles

EXPLORATION LESSON 38
DIRECTIONS PART 1:

1. Look at square #1. Measure one of the sides in inches. Label all four side lengths of the square.
2. Construct a diagonal.
3. Without measuring with your ruler, calculate the length of the diagonal.
4. Complete the same process for squares #2 and #3 and fill-in the table below.

	Side Length (leg of right triangle)	Exact Diagonal Length (hypotenuse)
Square #1		
Square #2		
Square #3		

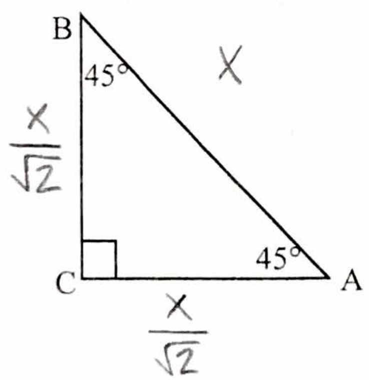
QUESTIONS PART 1:

1. Classify the triangles formed by drawing the diagonal of square by its sides and its angles.

2. What are the angle measures in the two triangles formed by the diagonal?

3. If you know the leg of a right triangle, how could you find the length of the hypotenuse without using Pythagorean Theorem or measuring? Look for a pattern within your table and then write an equation that relates the hypotenuse to the leg of the right triangle.

4. Based on relationships you have observed, label the sides with formulas of the following triangle using a lower case "L" for side AC.



GEOMETRY NOTES

LESSON 38: Special Right Triangles

DIRECTIONS PART 2:

1. The following triangles are equilateral. Measure the side length of Triangle #1 in inches. Label all 3 side lengths of the triangle.
2. Construct an altitude from one of the vertices.
3. Use Pythagorean Theorem to find the exact length (no decimal answers) of the altitude.
4. Complete the same process for triangles #2 and #3 and fill-in the table below.

	Side Length of Triangle ________	Altitude Length ________	Hypotenuse Length
Triangle #1			
Triangle #2			
Triangle #3			

QUESTIONS PART 2:

1. Classify the triangles formed by drawing the altitude by its sides and its angles.

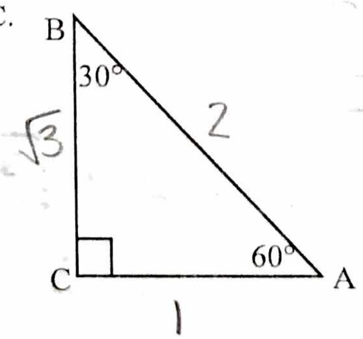
2. What are the angle measures in the two triangles formed by the altitude?

3. The side length of the triangle and the altitude are both legs forming the hypotenuse. Looking at your data, you should notice that one leg is always longer than the other. Label the longer one as "long leg" and the other as "short leg" in the blanks in the table above.

4. Look for a pattern within your table and then write an equation that relates the hypotenuse to the short leg.

5. Look for a pattern within your table and then write an equation that relates the long leg to the short leg.

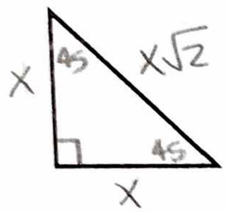
6. Based on relationships you have observed, label the sides with the formulas from above of the following triangle using "s" for side AC.

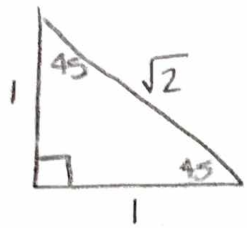


GEOMETRY NOTES

LESSON 38: Special Right Triangles

45°-45°-90° TRIANGLES
(FORMULA METHOD)

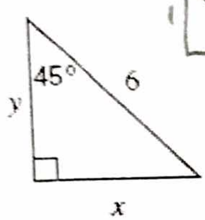




Formula: hypotenuse = leg • $\sqrt{2}$

EXAMPLES: Solve for the variables.

1.



$\frac{6}{\sqrt{2}} = \frac{x}{1}$

$6 = \sqrt{2} \cdot x$

$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$ $\boxed{x = 3\sqrt{2}}$ $\boxed{y = 3\sqrt{2}}$

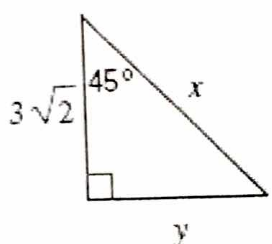
$\frac{6}{\sqrt{2}} = \frac{x \cdot \cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$

$x = 3\sqrt{2}$

$y = 3\sqrt{2}$

2.



$\boxed{y = 3\sqrt{2}}$

$\frac{3\sqrt{2}}{1} = \frac{x}{\sqrt{2}}$

$x = 3\sqrt{2} \cdot \sqrt{2}$

$x = 3 \cdot 2$

$\boxed{x = 6}$

$x = 3\sqrt{2} \cdot \sqrt{2}$

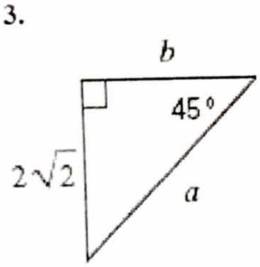
$x = 3 \cdot 2$

$x = 6$

$y = 3\sqrt{2}$

YOU TRY:

3.



$\boxed{b = 2\sqrt{2}}$

$\frac{2\sqrt{2}}{1} = \frac{a}{\sqrt{2}}$

$a = 2\sqrt{2} \cdot \sqrt{2}$

$a = 2 \cdot 2$

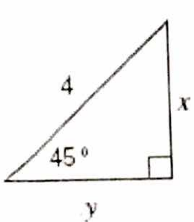
$\boxed{a = 4}$

$a = 2\sqrt{2} \cdot \sqrt{2}$

$a = 2 \cdot 2$

$a = 4$

4.



$\frac{4}{\sqrt{2}} = \frac{x}{1}$

$x\sqrt{2} = 4$

$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$

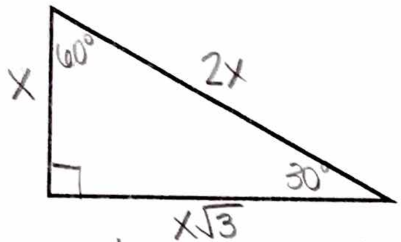
$\boxed{x = 2\sqrt{2}}$ $\boxed{y = 2\sqrt{2}}$

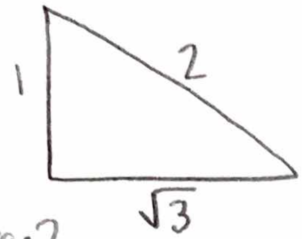
$\frac{4}{\sqrt{2}} = \frac{x \cdot \sqrt{2}}{\sqrt{2}}$

$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$

$x = 2\sqrt{2}$

30°-60°-90° Triangles
(FORMULA METHOD)

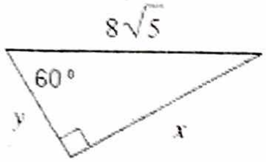




Formulas: hypotenuse = short leg · 2
long leg = short leg · $\sqrt{3}$

EXAMPLES: Solve for the variables.

5.



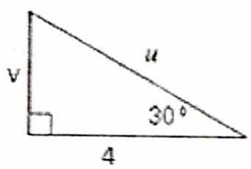
$\frac{8\sqrt{5}}{2} = \frac{y \cdot 2}{2}$

$\boxed{y = 4\sqrt{5}}$

$X = 4\sqrt{5} \cdot \sqrt{3}$

$\boxed{X = 4\sqrt{15}}$

6.



$\frac{4}{\sqrt{3}} = \frac{v \cdot \sqrt{3}}{\sqrt{3}}$

$v = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

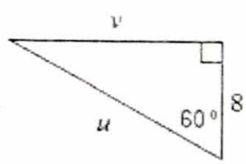
$\boxed{v = \frac{4\sqrt{3}}{3}}$

$U = \frac{4\sqrt{3}}{3} \cdot 2$

$\boxed{U = \frac{8\sqrt{3}}{3}}$

YOU TRY:

7.



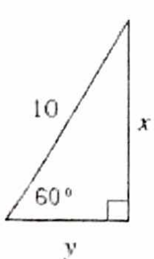
$V = 8 \cdot \sqrt{3}$

$\boxed{V = 8\sqrt{3}}$

$U = 8 \cdot 2$

$\boxed{U = 16}$

8.



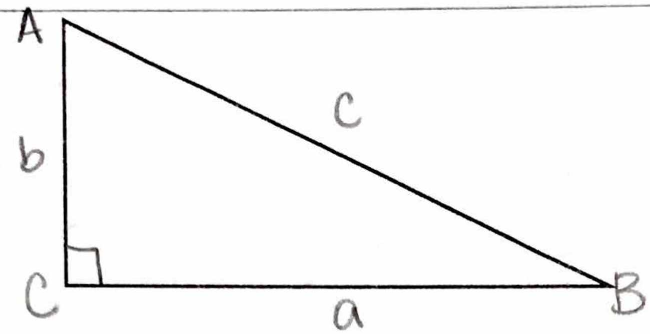
$\frac{10}{2} = \frac{y \cdot 2}{2}$

$\boxed{y = 5}$

$X = 5 \cdot \sqrt{3}$

$\boxed{X = 5\sqrt{3}}$

<u>Trigonometry:</u>



ONLY USE ON THE ACUTE ANGLES OF RIGHT TRIANGLES!

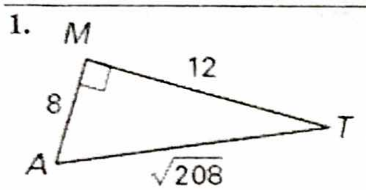
Sine A = $\frac{\text{opposite}}{\text{hypotenuse}}$ = $\frac{a}{c}$

Cosine A= $\frac{\text{adjacent}}{\text{hypotenuse}}$ = $\frac{b}{c}$

Tangent A= $\frac{\text{opposite}}{\text{adjacent}}$ = $\frac{a}{b}$

EXAMPLES: **Find the sine, cosine, and tangent of the acute angles of the triangle. Round answers to 4 decimals.**

1.



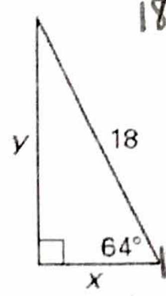
sin A = $\frac{12}{\sqrt{208}} = 0.8321$ sin T = $\frac{8}{\sqrt{208}} = 0.5547$

cos A = $\frac{8}{\sqrt{208}} = 0.5547$ cos T = $\frac{12}{\sqrt{208}} = 0.8321$

tan A = $\frac{12}{8} = 1.5$ tan T = $\frac{8}{12} = 0.\overline{6}$

EXAMPLES: **Find the value of each variable. Round your answer to the nearest tenth.**

2.



$18 \cdot \sin 64 = \frac{y}{18} \cdot 18$

$y = 18\sin 64$

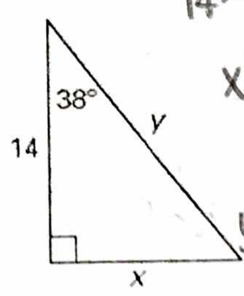
$\boxed{y = 16.2}$

$18 \cdot \cos 64 = \frac{x}{18} \cdot 18$

$x = 18\cos 64$

$\boxed{x = 7.9}$

3.



$14 \cdot \tan 38 = \frac{x}{14} \cdot 14$

$x = 14\tan 38$

$\boxed{x = 10.9}$

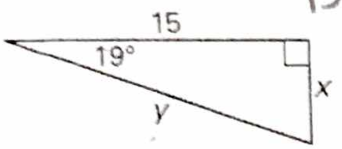
$y \cdot \cos 38 = \frac{14}{y} \cdot y$

$\frac{y\cancel{\cos 38}}{\cancel{\cos 38}} = \frac{14}{\cos 38}$

$\boxed{y = 17.8}$

YOU TRY

4.



$15 \cdot \tan 19 = \frac{x}{15} \cdot 15$

$x = 15\tan 19$

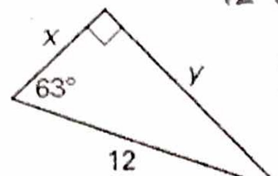
$\boxed{x = 5.2}$

$y \cdot \cos 19 = \frac{15}{y} \cdot y$

$\frac{y\cos 19}{\cos 19} = \frac{15}{\cos 19}$

$\boxed{y = 15.9}$

5.



$12 \cdot \sin 63 = \frac{y}{12} \cdot 12$

$y = 12\sin 63$

$\boxed{y = 10.7}$

$12 \cdot \cos 63 = \frac{x}{12} \cdot 12$

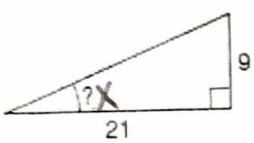
$x = 12\cos 63$

$\boxed{x = 5.4}$

We have used to trigonometry to find the length of sides. However, you can also use trigonometry to help you find the measures of angles.

EXAMPLES: Find the measure of the indicated angle to the nearest whole degree.

6.

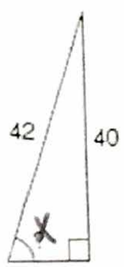


$\tan x = \frac{9}{21}$

$x = \tan^{-1}\left(\frac{9}{21}\right)$

$\boxed{x = 23.2°}$

7.

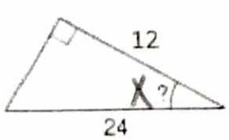


$\sin x = \frac{40}{42}$

$x = \sin^{-1}\left(\frac{40}{42}\right)$

$\boxed{x = 72.2}$

8.



$\cos x = \frac{12}{24}$

$x = \cos^{-1}\left(\frac{12}{24}\right)$

$\boxed{x = 60}$

GEOMETRY NOTES
LESSON 40: Solving Right Triangles

SOLVING RIGHT TRIANGLES

To solve a right triangle you need to know the following pieces of information:

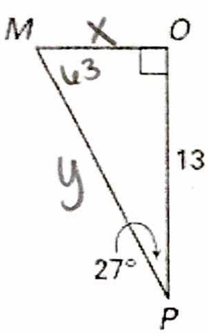
1. one acute angle & one side
2. two sides

You can solve a right triangle using the following methods from your triangle toolkit.

1. Geometric Mean
2. Pythagorean Theorem
3. Special Right Triangles (30,60,90) (45,45,90)
4. Trigonometry

EXAMPLES: Solve each triangle. Round decimals to nearest tenth.

1.



$13 \cdot \tan 27 = \frac{x}{13} \cdot 13$

$x = 13\tan 27$

$x = 6.6$

$13^2 + 6.6^2 = y^2$

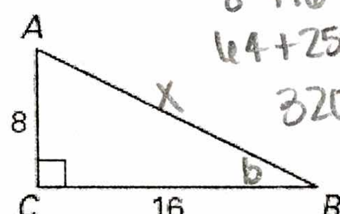
$169 + 43.56 = y^2$

$212.56 = y^2$

$y = 14.6$

$\boxed{MO = 6.6 \quad MP = 14.6 \quad \angle M = 63°}$

2.



$8^2 + 16^2 = x^2$

$64 + 256 = x^2$

$320 = x^2$

$x = 17.9$

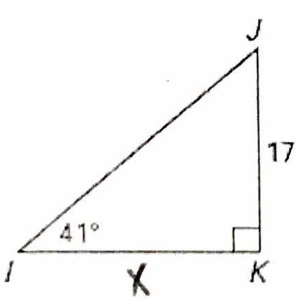
$\tan b = \frac{8}{16}$

$b = \tan^{-1}\left(\frac{8}{16}\right)$

$b = 26.6$

$\boxed{AB = 17.9 \quad \angle A = 63.4° \quad \angle B = 26.6°}$

3.



$\tan 41 = \frac{17}{x}$

$\frac{x\tan 41}{\tan 41} = \frac{17}{\tan 41}$

$x = 19.6$

$17^2 + 19.6^2 = y^2$

$289 + 384.16 = y^2$

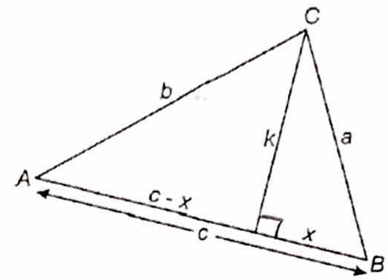
$673.16 = y^2$

$y = 25.9$

$\boxed{IK = 19.6 \quad IJ = 25.9 \quad \angle J = 49°}$

GEOMETRY NOTES
LESSON 41: Law of Sines and Cosines

LAW OF COSINES



1. The altitude k separates ΔABC into two right triangles. Use the Pythagorean Theorem to write two equations, one relating k, b, and c – x, and another relating a, k, and x.

$k^2+(c-x)^2=b^2$ $\quad x^2+k^2=a^2$

2. Notice that both equations contain k^2. (Why?) Solve each equation for k^2.

$k^2=b^2-(c-x)^2$ $\quad k^2=a^2-x^2$

3. Since both of the equations in Question 2 are equal to k^2, they can be set equal to each other. (Why is this true?) Set the equations equal to each other to form a new equation.

$b^2-(c-x)^2=a^2-x^2$

4. Notice that the equation in Question 3 involves x. However x is only PART of a side of ΔABC. Therefore, we will attempt to rewrite the equation in Question 3 so that it does not include x. Begin by expanding (hint: FOIL) the quantity $(c-x)^2$.

$b^2-(c^2-2cx+x^2)=a^2-x^2$ $\quad a^2=b^2-c^2+2cx$

$b^2-c^2+2cx-x^2=a^2-x^2$

5. Solve the equation in Question 4 for b^2.

$b^2=a^2+c^2-2cx$

6. The equation in Question 5 still involves x. To eliminate x from the equation, we will attempt to substitute an equivalent expression for x. Write an equation involving both cosB and x.

$\cos B=\frac{x}{a}$

7. Solve the equation from Question 6 for x. (Why solve for x?)

$x=a\cos B$

8. Substitute the equivalent expression for x into the equation from Question 5. The resulting equation contains only sides and angles of ΔABC. This equation is called **LAW OF COSINES.**

$b^2=a^2+c^2-2ca\cos B$

9. Using a similar method, two other forms of this law could be developed for a^2 and c^2. Based on all your work for Questions 1 – 7 and your LAW OF COSINES formula from Question 8, write the other two forms of the LAW OF COSINES for ΔABC.

$a^2=b^2+c^2-2bc\cos A$

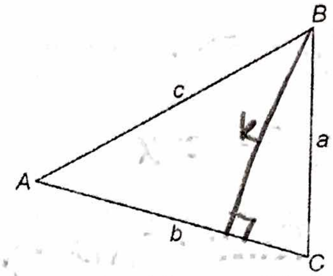
$c^2=a^2+b^2-2ab\cos C$

GEOMETRY NOTES

LESSON 41: Law of Sines and Cosines

LAW OF SINES

1. Sketch an altitude from vertex B on ΔABC to the right. Label it k.



2. The altitude, k, creates two right triangles inside ΔABC. Notice that <A is contained in one of the right triangles and <C is contained in the other. Using right triangle trigonometry, write two equations, one involving sinA and one involving sinC.

sinA = $\frac{k}{c}$ sinC = $\frac{k}{a}$

3. Notice that each of the equations in Question 2 involves k. (Why does this happen?) Solve each equation for k.

k is opposite of <A and <C

4. Since both equations in Question 4 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

$c\sin A = k$ $a\sin C = k$

$c\sin A = a\sin C$

5. Notice that the equation in Question 4 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 4, regrouping a with sinA and c with sinC (hint: it should look like a proportion).

$\frac{\cancel{c}\sin A}{\cancel{c}a} = \frac{a\sin C}{\cancel{a}c}$ $\frac{\sin A}{a} = \frac{\sin C}{c}$

6. Using the proportion from Question 5, add on another ratio that involves b and sinB. Together Question 5 and Question 6 combine to form an equation called **LAW OF SINES**. Write the equation for the **LAW OF SINES**.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

GEOMETRY NOTES
LESSON 41: Law of Sines and Cosines

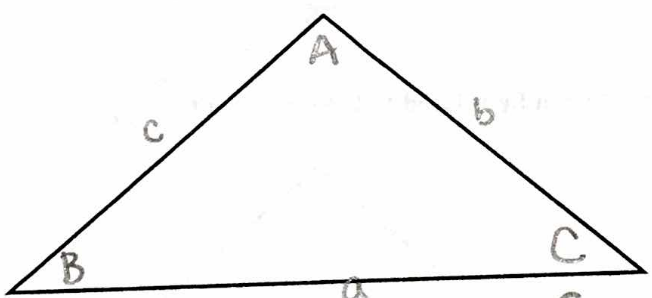
You can solve a right triangle using the following methods from your triangle toolkit.

1. Geometric Mean
2. Pythagorean Theorem
3. Special Right Triangles (30,60,90) (45,45,90)
4. Trigonometry

To solve an oblique triangle (a non-right triangle) there are 2 methods:

1. Law of Sines
2. Law of Cosines

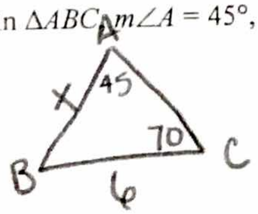
LAW OF SINES
In any triangle the ratio of the sine of an angle to the length of its opposite side is constant. Therefore the following relationship is true:



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

EXAMPLES: Use the Law of Sines and the information given to solve. Draw a picture first.

1. In ΔABC, $m\angle A = 45°$, $m\angle C = 70°$, and $BC = 6$. To the nearest tenth, what is AB?



$$\frac{\sin 45}{6} = \frac{\sin 70}{x}$$

$$\frac{x\cancel{\sin 45}}{\cancel{\sin 45}} = \frac{6\sin 70}{\sin 45}$$

$$x = \frac{6\sin 70}{\sin 45}$$

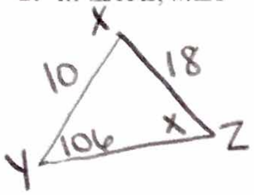
$$x = 7.97$$

$$\boxed{AB = 7.97}$$

GEOMETRY NOTES

LESSON 41: Law of Sines and Cosines

2. In $\triangle XYZ$, $m\angle Y = 106°$, $XY = 10$, and $XZ = 18$. To the nearest tenth, what is $m\angle Z$?



$$\frac{\sin 106}{18} = \frac{\sin x}{10}$$

$$18 \sin x = 10 \sin 106$$

$$\sin x = \frac{10 \sin 106}{18}$$

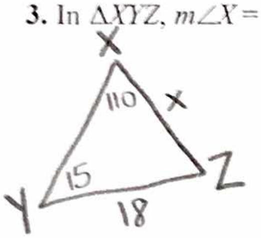
$$x = \sin^{-1}\left(\frac{10 \sin 106}{18}\right)$$

$$x = 32.3$$

$$\boxed{m\angle Z = 32.3°}$$

YOU TRY:

3. In $\triangle XYZ$, $m\angle X = 110°$, $m\angle Y = 15°$, and $YZ = 18$. To the nearest tenth, what is XZ?



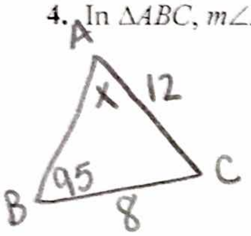
$$\frac{\sin 110}{18} = \frac{\sin 15}{x}$$

$$\frac{x \cancel{\sin 110}}{\cancel{\sin 110}} = \frac{18 \sin 15}{\sin 110}$$

$$x = \frac{18 \sin 15}{\sin 110}$$

$$\boxed{x = 26.8}$$

4. In $\triangle ABC$, $m\angle B = 95°$, $AC = 12$, and $BC = 8$. To the nearest tenth, what is $m\angle A$?



$$\frac{\sin 95}{12} = \frac{\sin x}{8}$$

$$\frac{\cancel{12} \sin x}{\cancel{12}} = \frac{8 \sin 95}{12}$$

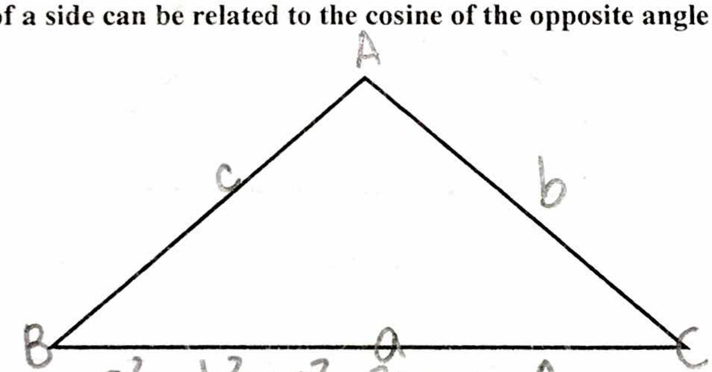
$$\sin x = \frac{8 \sin 95}{12}$$

$$x = \sin^{-1}\left(\frac{8 \sin 95}{12}\right)$$

$$\boxed{x = 41.6}$$

LAW OF COSINES

Given a triangle, the length of a side can be related to the cosine of the opposite angle and the lengths of the two other sides.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

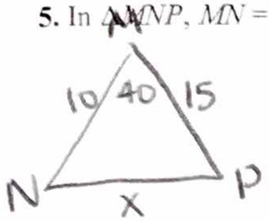
$$c^2 = a^2 + b^2 - 2ab \cos C$$

GEOMETRY NOTES

LESSON 41: Law of Sines and Cosines

EXAMPLES: **Use the Law of Cosines and the information given to solve. Draw a picture first.**

5. In $\triangle MNP$, $MN = 10$, $MP = 15$, and $m\angle M = 40°$. To the nearest tenth, what is NP?



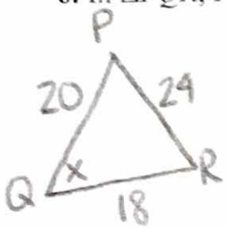
$X^2 = 10^2 + 15^2 - 2 \cdot 10 \cdot 15 \cos 40$

$X^2 = 100 + 225 - 300 \cos 40$

$X^2 = 95.186$

$\boxed{X = 9.76}$

6. In $\triangle PQR$, $PQ = 20$, $QR = 18$, and $PR = 24$. To the nearest tenth, what is $m\angle Q$?



$24^2 = 18^2 + 20^2 - 2 \cdot 18 \cdot 20 \cos X$

$576 = 324 + 400 - 720 \cos X$

$576 = 724 - 720 \cos X$

$\frac{-148}{-720} = \frac{-\cancel{720} \cos X}{-\cancel{720}}$

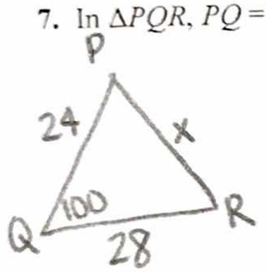
$\cos X = \frac{148}{720}$

$X = \cos^{-1}\left(\frac{148}{720}\right)$

$\boxed{X = 78.1°}$

YOU TRY:

7. In $\triangle PQR$, $PQ = 24$, $QR = 28$, and $m\angle Q = 100°$. To the nearest tenth, what is PR?



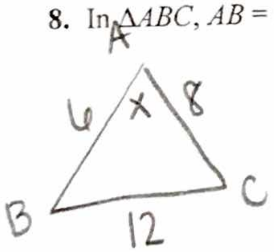
$X^2 = 24^2 + 28^2 - 2 \cdot 24 \cdot 28 \cos 100$

$X^2 = 576 + 784 - -233.38$

$X^2 = 1593.38$

$\boxed{X = 39.9}$

8. In $\triangle ABC$, $AB = 6$, $BC = 12$, and $AC = 8$. To the nearest tenth, what is $m\angle A$?



$12^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cos X$

$144 = 36 + 64 - 2 \cdot 6 \cdot 8 \cos X$

$144 = 100 - 96 \cos X$

$\frac{44}{-96} = \frac{-\cancel{96} \cos X}{-\cancel{96}}$

$\cos X = \frac{44}{-96}$

$X = \cos^{-1}\left(\frac{44}{-96}\right)$

$\boxed{X = 117.28°}$