










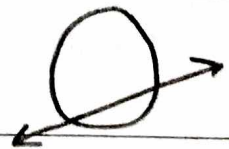


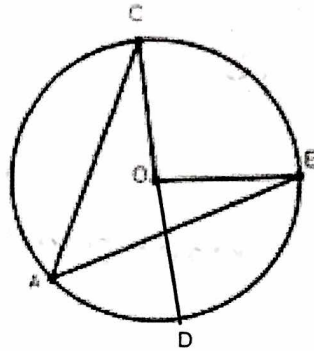
Lessons 43-47 Vocabulary Words

Name	Definition	Picture/Example
Radius	<ul style="list-style-type: none"> • Segment with one endpoint on circle + one endpoint in center - distance from center to any point on circle 	
Diameter	<ul style="list-style-type: none"> • Segment passes through center w/ both endpoints on circle - distance across circle 	
Semicircle	<ul style="list-style-type: none"> • arc formed by a diameter - measure is exactly 180° 	
Minor Arc	<ul style="list-style-type: none"> • arc formed by central angle whose measure is less than 180° 	
Major Arc	<ul style="list-style-type: none"> • remaining arc left over from a minor arc whose measure is greater than 180° but less than 360° 	
Central Angle	<ul style="list-style-type: none"> angle formed by two radii in the circle - vertex is center 	
Chord	<ul style="list-style-type: none"> Segments with endpoints on circle 	
Point of Tangency	<ul style="list-style-type: none"> point where the tangent touches the circle 	
Tangent	<ul style="list-style-type: none"> line that intersects a circle at one distinct point 	
Inscribed Angle	<ul style="list-style-type: none"> angle formed by two chords - vertex on circle 	
Intercepted Arc	<ul style="list-style-type: none"> part of arc formed by inscribed angle 	
Secant	<ul style="list-style-type: none"> line that intersects a circle at two distinct points (passes through circle) 	

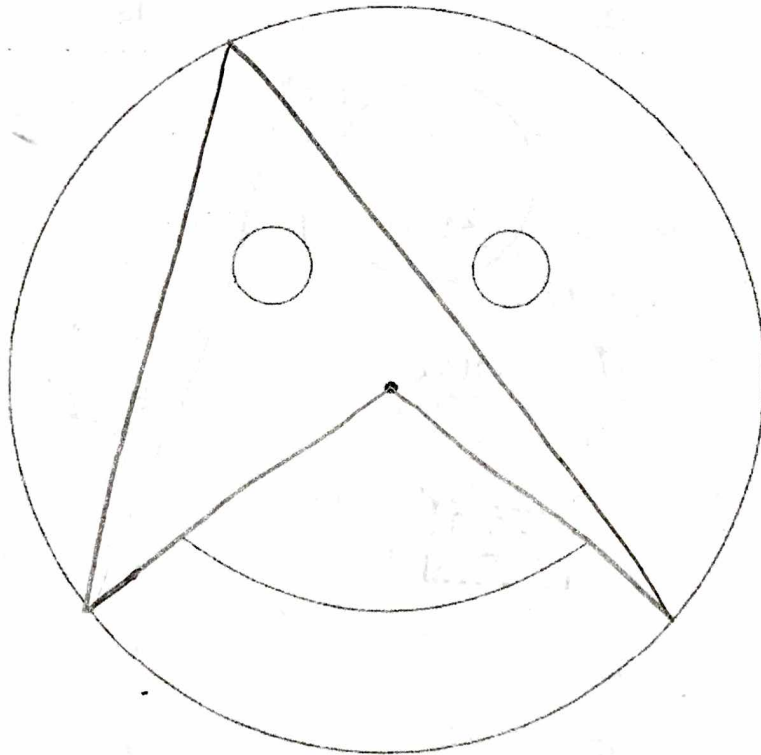
GEOMETRY NOTES
LESSON 43: Angles and Arcs

EXAMPLES: Use the diagram to respond to the following statements:

1. Name a central angle. $\angle COB, \angle BOD$
2. Name a minor arc. $\widehat{CB}, \widehat{BD}, \widehat{DA}, \widehat{AC}$
3. Name a major arc. $\widehat{CAB}, \widehat{BAD}$
4. Name a semi-circle. $\widehat{CBD}, \widehat{CAD}$
5. Name an inscribed angle. $\angle CAB$
6. Name an intercepted arc. \widehat{CB}



EXPLORATION: Central Angle, Minor Arc, Major Arc, Intercepted Arc, and Inscribed Angle Relationships



Based on your exploration, determine the relationships below:

CENTRAL ANGLE = MINOR ARC

MAJOR ARC = 360° - minor arc

SEMI-CIRCLE = 180°

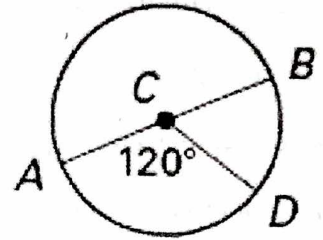
INSCRIBED ANGLE = $\frac{1}{2}$ intercepted arc

INTERCEPTED ARC = $2 \cdot$ inscribed angle

GEOMETRY NOTES
LESSON 43: Angles and Arcs

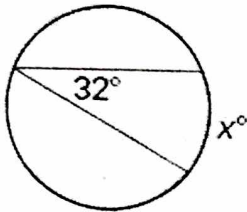
EXAMPLES: Name and find the measure of each arc of circle C.

7. arc AD \widehat{AD}
 120° minor arc
8. arc ADB \widehat{ADB}
 180° semicircle
9. arc DBA \widehat{DBA}
 $360 - 120 = 240^\circ$ major arc
10. arc BD \widehat{BD}
 60° minor arc



EXAMPLES: Find the value of x.

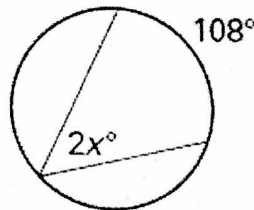
11.



$$x = 32 \cdot 2$$

$$\boxed{x = 64}$$

12.

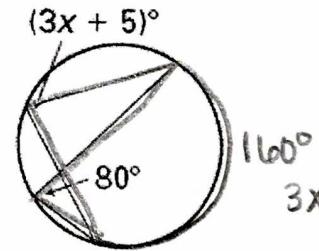


$$2(2x) = 108$$

$$4x = 108$$

$$\boxed{x = 27}$$

13.



$$2(3x + 5) = 160$$

$$6x + 10 = 160$$

$$6x = 150$$

$$\boxed{x = 25}$$

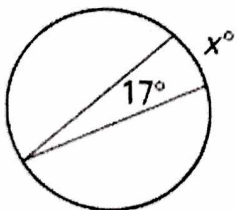
$$3x + 5 = 86$$

$$3x = 75$$

$$x = 25$$

YOU TRY:

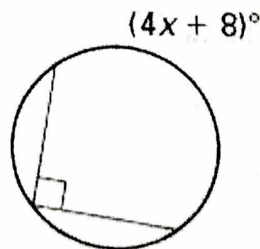
14.



$$x = 2 \cdot 17$$

$$\boxed{x = 34}$$

15.



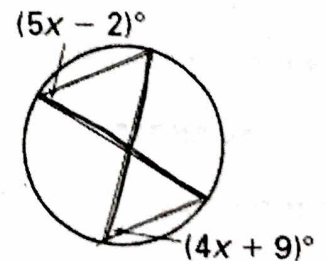
$$2 \cdot 90 = 4x + 8$$

$$180 = 4x + 8$$

$$172 = 4x$$

$$\boxed{x = 43}$$

16.



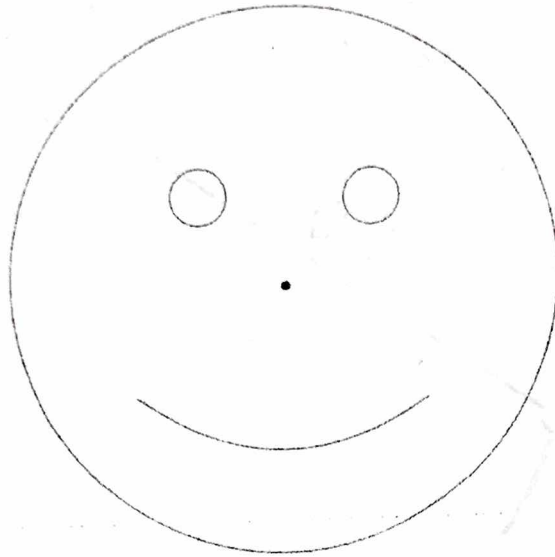
$$5x - 2 = 4x + 9$$

$$x - 2 = 9$$

$$\boxed{x = 11}$$

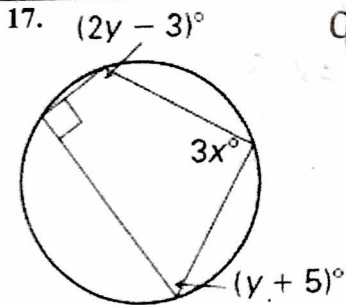
GEOMETRY NOTES
LESSON 44: Inscribed Quadrilaterals

EXPLORATION: Inscribed Quadrilaterals



For any inscribed quadrilateral the opposite angles are supplementary.

EXAMPLES: Find the value of each variable.



$$90 + 3x = 180$$

$$3x = 90$$

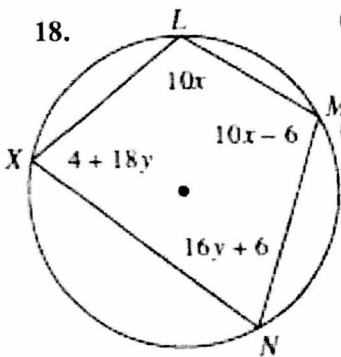
$$\boxed{x = 3}$$

$$2y - 3 + y + 5 = 180$$

$$3y + 2 = 180$$

$$3y = 178$$

$$\boxed{y = 59.3}$$



$$\textcircled{1} 10x + 16y + 6 = 180$$

$$10x + 16y = 174$$

$$\textcircled{2} 10x - 6 + 18y + 4 = 180$$

$$10x + 18y - 2 = 180$$

$$10x + 18y = 182$$

$$10x + 16y = 174$$

$$-(10x + 18y = 182)$$

$$\hline 10x + 16y = 174$$

$$-10x - 18y = -182$$

$$\hline -2y = -8$$

$$\boxed{y = 4}$$

$$10x + 16(4) + 6 = 180$$

$$10x + 64 + 6 = 180$$

$$10x + 70 = 180$$

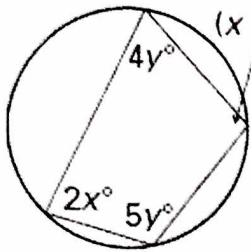
$$10x = 110$$

$$\boxed{x = 11}$$

GEOMETRY NOTES
LESSON 44: Inscribed Quadrilaterals

YOU TRY:

19.



$$2x + x + 12 = 180$$

$$3x + 12 = 180$$

$$3x = 168$$

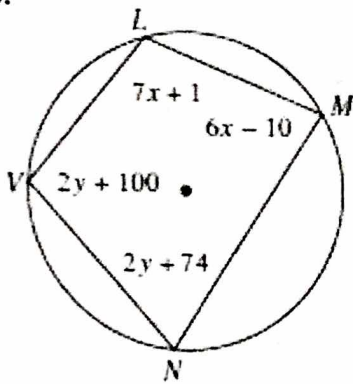
$$\boxed{x = 56}$$

$$5y + 4y = 180$$

$$9y = 180$$

$$\boxed{y = 20}$$

20.



$$\textcircled{1} 7x + 1 + 2y + 74 = 180$$

$$7x + 2y + 75 = 180$$

$$7x + 2y = 105$$

$$\textcircled{2} 6x - 10 + 2y + 100 = 180$$

$$6x + 2y + 90 = 180$$

$$6x + 2y = 90$$

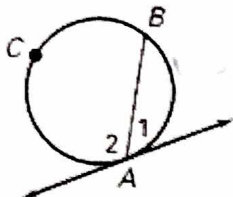
$$\begin{array}{r} 7x + 2y = 105 \\ - (6x + 2y = 90) \\ \hline \end{array}$$

$$\begin{array}{r} 7x + 2y = 105 \\ - 6x - 2y = -90 \\ \hline \end{array}$$

$$\boxed{x = 15}$$

GEOMETRY NOTES
LESSON 45: Angle Relationships in Circles

ANGLE "ON" CIRCLE (Angles formed when a tangent and a chord intersect at a point on the circle):



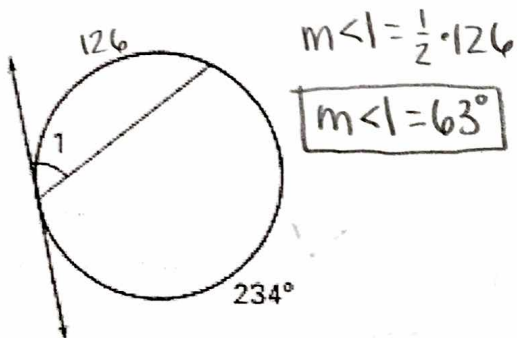
The measure of the angle is half of the intercepted arc

$$\angle 1 = \frac{1}{2}(\widehat{AB})$$

$$\angle 2 = \frac{1}{2}(\widehat{BCA})$$

EXAMPLES: Find the measure of the indicated arc or angle.

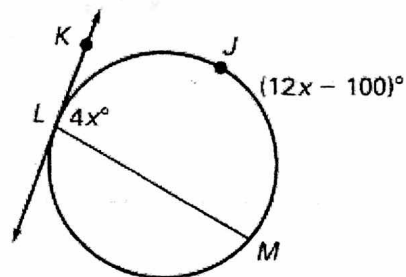
1. $m\angle 1$



$$m\angle 1 = \frac{1}{2} \cdot 126$$

$$m\angle 1 = 63^\circ$$

2. $m\angle KLM$



$$4x = \frac{1}{2}(12x - 100)$$

$$4x = 6x - 50$$

$$-2x = -50$$

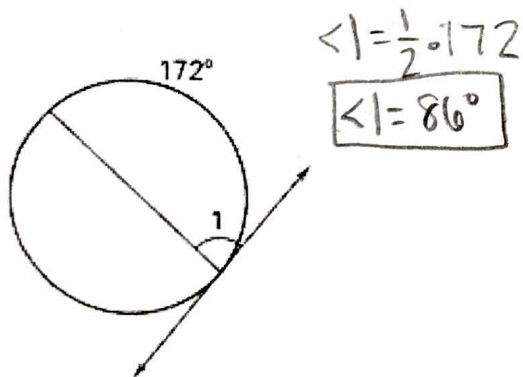
$$x = 25$$

$$m\angle KLM = 4 \cdot 25$$

$$m\angle KLM = 100^\circ$$

YOU TRY:

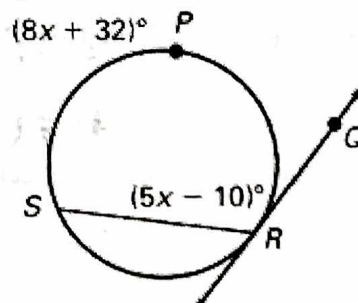
3. $m\angle 1$



$$\angle 1 = \frac{1}{2} \cdot 172$$

$$\angle 1 = 86^\circ$$

4. $m\angle QRS$



$$5x - 10 = \frac{1}{2}(8x + 32)$$

$$5x - 10 = 4x + 16$$

$$x - 10 = 16$$

$$x = 26$$

$$m\angle QRS = 5(26) - 10$$

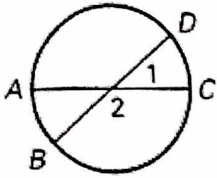
$$m\angle QRS = 130 - 10$$

$$m\angle QRS = 120^\circ$$

GEOMETRY NOTES

LESSON 45: Angle Relationships in Circles

ANGLE "INSIDE" CIRCLE (Angles formed when two chords intersect at point inside the circle):

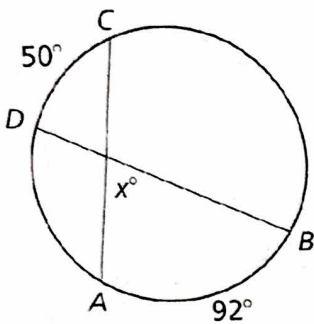


$$\angle 1 = \frac{1}{2}(\widehat{CD} + \widehat{AB})$$

$$\angle 2 = \frac{1}{2}(\widehat{BC} + \widehat{AD})$$

EXAMPLES: Find the value of the variable.

5.

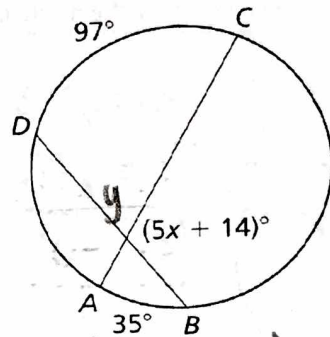


$$x = \frac{1}{2}(50 + 92)$$

$$x = \frac{1}{2}(142)$$

$$\boxed{x = 71}$$

6.



$$y = \frac{1}{2}(97 + 35)$$

$$y = \frac{1}{2}(132)$$

$$y = 66$$

$$5x + 14 + 66 = 180$$

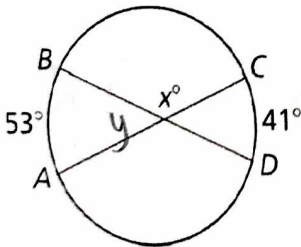
$$5x + 80 = 180$$

$$5x = 100$$

$$\boxed{x = 20}$$

YOU TRY:

7.



$$y = \frac{1}{2}(53 + 41)$$

$$y = \frac{1}{2}(94)$$

$$y = 47$$

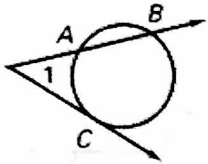
$$x + 47 = 180$$

$$\boxed{x = 133}$$

GEOMETRY NOTES
LESSON 45: Angle Relationships in Circles

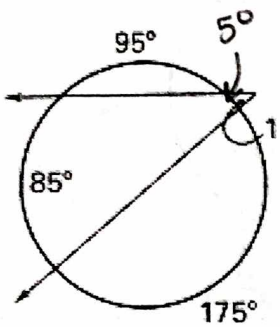
ANGLE "OUTSIDE" CIRCLE (Angles formed from two tangent lines or two secant lines intersect outside the circle):

$$\angle = \frac{1}{2}(\widehat{BC} - \widehat{AC})$$



EXAMPLES: Find the value of the angle or the variable.

8. $m\angle 1$

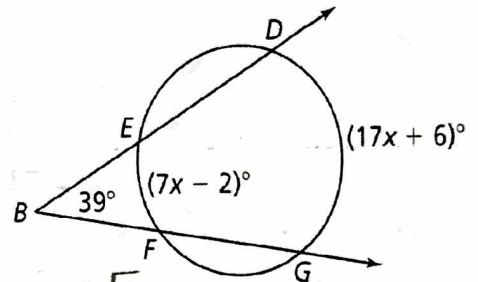


$$\angle 1 = \frac{1}{2}(85 - 5)$$

$$\angle 1 = \frac{1}{2}(80)$$

$$\angle 1 = 40^\circ$$

9.



$$39 = \frac{1}{2}[(17x + 6) - (7x - 2)]$$

$$39 = \frac{1}{2}(10x + 8)$$

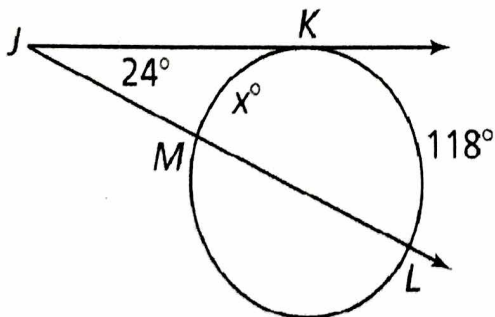
$$39 = 5x + 4$$

$$35 = 5x$$

$$x = 7$$

YOU TRY:

10.



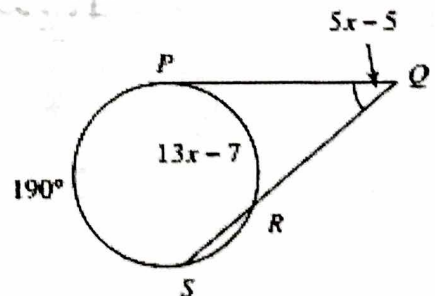
$$24 = \frac{1}{2}(118 - x)$$

$$48 = 118 - x$$

$$-70 = -x$$

$$x = 70$$

11.



$$5x - 5 = \frac{1}{2}(190 - (13x - 7))$$

$$5x - 5 = \frac{1}{2}(-13x + 197)$$

$$2(5x - 5) = -13x + 197$$

$$10x - 10 = -13x + 197$$

$$23x = 207$$

$$x = 9$$

GEOMETRY NOTES

LESSON 46: Parts of a Circle and Tangents to Circle

EXAMPLES: Identify each segment.

1. \overline{DG} tangent

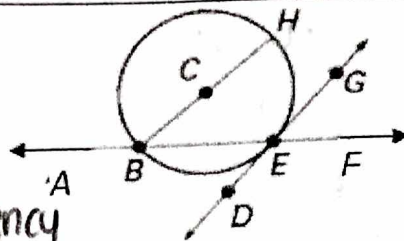
2. \overline{BE} chord

3. \overline{AF} secant

4. \overline{BH} diameter

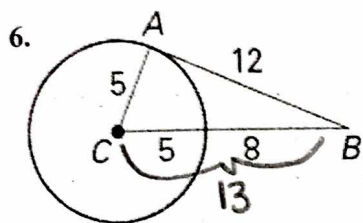
5. \overline{CH} radius

6. E point of tangency



If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

EXAMPLES: Tell whether AB is tangent to circle C. Explain your reasoning.



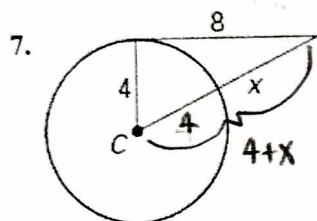
$$5^2 + 12^2 = 13^2$$

$$25 + 144 = 169$$

$$169 = 169 \checkmark$$

tangent

EXAMPLES: Find the value of x. Assume that the segments that appear to be tangent are tangent.



$$4^2 + 8^2 = (x+4)^2$$

$$16 + 64 = x^2 + 8x + 16$$

$$80 = x^2 + 8x + 16$$

$$0 = x^2 + 8x - 64$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-64)}}{2}$$

$$x = \frac{-8 \pm \sqrt{64 + 256}}{2}$$

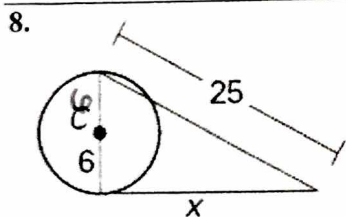
$$x = \frac{-8 \pm \sqrt{320}}{2}$$

$x = 4.94$

OR

$x = -12.94$

YOU TRY:



$$12^2 + x^2 = 25^2$$

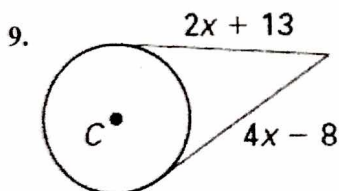
$$144 + x^2 = 625$$

$$x^2 = 481$$

$x = 21.93$

If two tangent lines intersect outside, then they are congruent.

EXAMPLES: Find the value of x



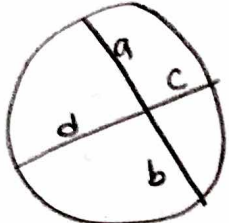
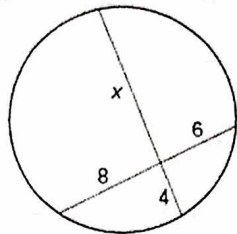
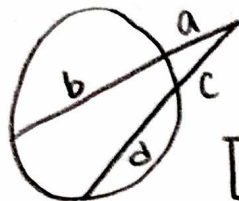
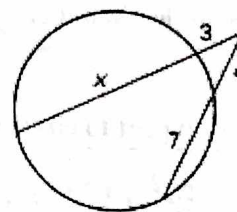
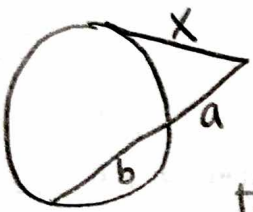
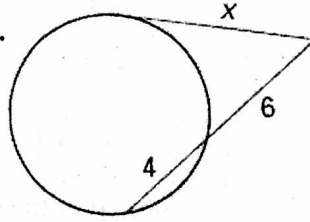
$$2x + 13 = 4x - 8$$

$$13 = 2x - 8$$

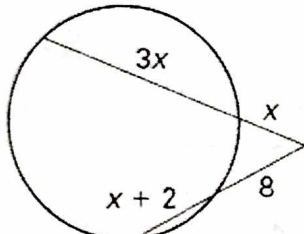
$$21 = 2x$$

$x = 10.5$

GEOMETRY NOTES
LESSON 47: Segment Lengths in Circles

Finding Segment Lengths in Circles	Examples
<p>1.  2 chords $ab = cd$ part · part</p>	<p>1.  $4x = 6 \cdot 8$ $4x = 48$ $x = 12$</p>
<p>2.  2 secants $a(a+b) = c(c+d)$ outside · whole</p>	<p>2.  $3(3+x) = 4 \cdot 11$ $9+3x = 44$ $3x = 35$ $x = 11.\bar{6}$</p>
<p>3.  tangent + secant $x \cdot x = a(a+b)$ outside · whole</p>	<p>3.  $x \cdot x = 6 \cdot 10$ $x^2 = 60$ $x = \sqrt{60}$ $x = 7.75$</p>

EXAMPLES: Find x.

4.  $x(4x) = 8(x+10)$
 $4x^2 = 8x + 80$
 $0 = 4x^2 - 8x - 80$

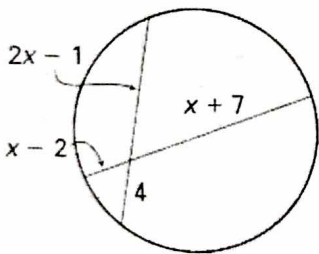
$$x = \frac{+8 \pm \sqrt{(-8)^2 - 4(4 \cdot -80)}}{8}$$

$$x = \frac{8 \pm \sqrt{1344}}{8}$$

$$x = 1 \pm 4.58$$

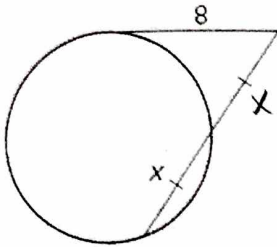
$$\boxed{x = 5.58}$$

OR
 $x = -3.58$

5.  $4(2x-1) = (x-2)(x+7)$
 $8x-4 = x^2+5x-14$
 $0 = x^2-3x-10$
 $0 = (x-5)(x+2)$
 $0 = x-5$ $0 = x+2$
 $x = 5$ $x = -2$

GEOMETRY NOTES
LESSON 47: Segment Lengths in Circles

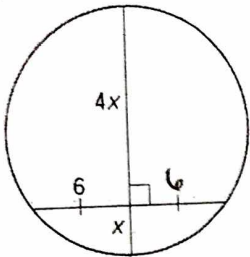
6.



$$\begin{aligned} 8 \cdot 8 &= x(2x) \\ 64 &= 2x^2 \\ x^2 &= 32 \\ \boxed{x &= 5.66} \end{aligned}$$

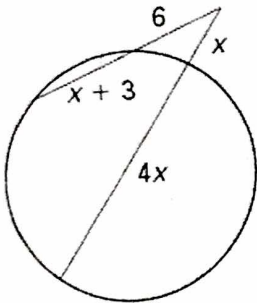
YOU TRY:

7.



$$\begin{aligned} 4x \cdot x &= 6 \cdot 6 \\ 4x^2 &= 36 \\ x^2 &= 9 \\ \boxed{x &= 3} \end{aligned}$$

8.



$$\begin{aligned} 6(x+9) &= x(5x) \\ 6x+54 &= 5x^2 \\ 0 &= 5x^2 - 6x - 54 \end{aligned}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(5)(-54)}}{10}$$

$$x = \frac{6 \pm \sqrt{1116}}{10}$$

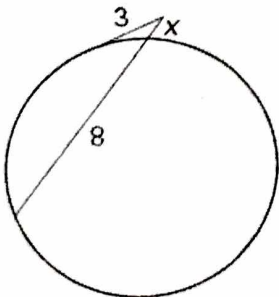
$$x = 0.6 \pm 3.34$$

$$\boxed{x = 3.94}$$

OK

$$x = -3.34$$

9.



$$\begin{aligned} 3 \cdot 3 &= x(x+8) \\ 9 &= x^2 + 8x \\ 0 &= x^2 + 8x - 9 \\ 0 &= (x+9)(x-1) \\ 0 &= x+9 & 0 &= x-1 \\ x &= -9 & \boxed{x &= 1} \end{aligned}$$