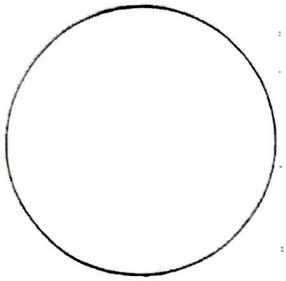
GEOMETRY NOTES

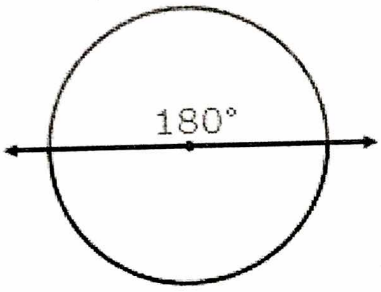
LESSON 48: Circumference/Arc Length and Areas of Circles/Sectors

EXPLORATION PART ONE:

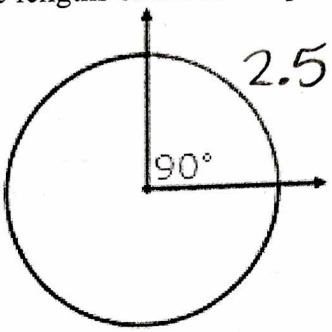
The circumference of the circle below is 10cm.

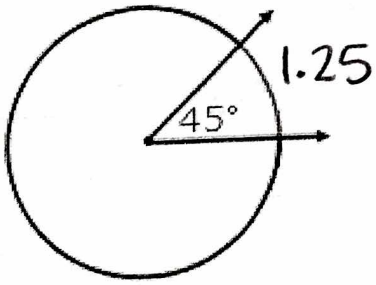


Based on the information above, what are the lengths of the intercepted arcs for the circles pictured below (not the angle measure)?



5





CIRCUMFERENCE/ARC LENGTH:

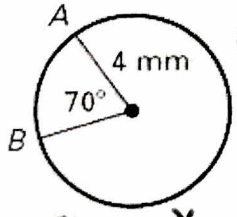
Circumference: $C=2\pi r$ OR $C=\pi d$ Twinkle twinkle

Find the circumference of a circle with a diameter of 16 cm. Leave answer with π.

Circumference and Arc Length: $\frac{\text{degree of arc}}{360} = \frac{\text{arc length}}{\text{circumference}}$

EXAMPLES: Find the indicated measure.

1. arc AB



$C=2\pi \cdot 4$

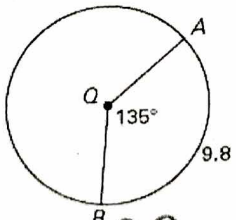
$C=8\pi$

$\frac{70}{360} = \frac{x}{8\pi}$

$\frac{360x}{360} = \frac{560\pi}{360}$

$x=4.89$

2. Circumference

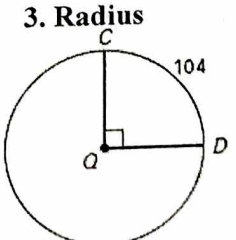


$\frac{135}{360} = \frac{9.8}{x}$

$\frac{135x}{135} = \frac{3528}{135}$

$x=26.13$

3. Radius



$\frac{90}{360} = \frac{104}{x}$

$90x = 37440$

$x=416$

$C=416$

$416=2\pi r$

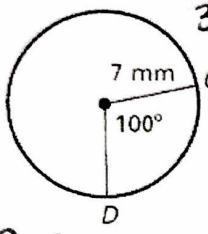
$r=66.21$

GEOMETRY NOTES

LESSON 48: Circumference/Arc Length and Areas of Circles/Sectors

YOU TRY:

4. arc CD



$\frac{100}{360} = \frac{x}{14\pi}$

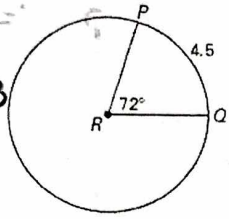
$360x = 4398.23$

$x = 12.22$

$C = 2\pi \cdot 7$

$C = 14\pi$

5. Circumference

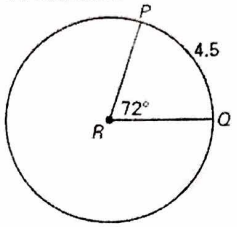


$\frac{72}{360} = \frac{4.5}{C}$

$72C = 1620$

$C = 22.5$

6. Radius

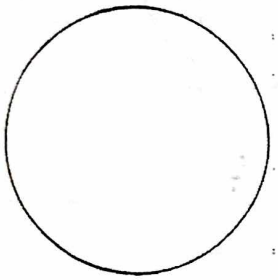


$\frac{22.5}{2\pi} = \frac{2\pi r}{2\pi}$

$r = 3.58$

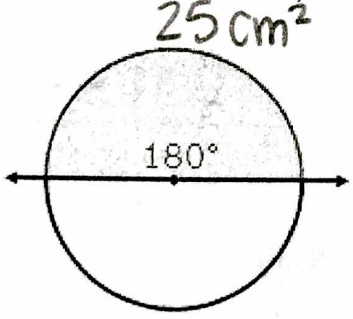
EXPLORATION PART TWO:

The area of the circle below is $50cm^2$.

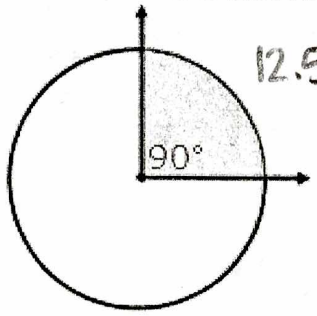


$A = 50cm^2$

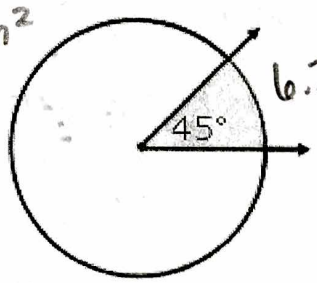
Based on the information above, what are the areas of the shaded regions for the circles pictured below?



$25\,cm^2$



$12.5cm^2$



$6.25cm^2$

GEOMETRY NOTES

LESSON 48: Circumference/Arc Length and Areas of Circles/Sectors

AREA OF A CIRCLE/AREA OF SECTOR:

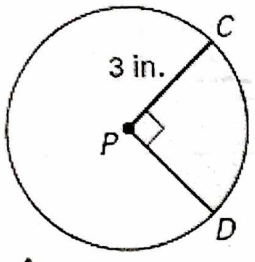
Area: πr^2 Fuzzy Wuzzy

Find the area of a circle with a radius of 5. Leave answer with π. $\pi 5^2 = 25\pi$

Area of a Circle and Area of a Sector: $\frac{\text{degree of arc}}{360} = \frac{\text{area of sector}}{\text{area}}$

EXAMPLES: Find the area of the shaded region

7.



$A = 9\pi$

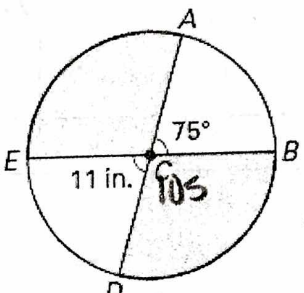
$\frac{90}{360} = \frac{x}{9\pi}$

$360x = 90 \cdot 9\pi$

$\frac{360x}{360} = \frac{2544.69}{360}$

$x = 7.07$

8.



$A = 11^2\pi$

$\frac{105}{360} = \frac{x}{121\pi}$

$360x = 105 \cdot 121\pi$

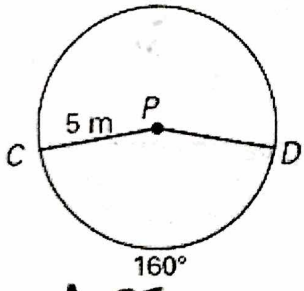
$\frac{360x}{360} = \frac{39913.93}{360}$

$x = 110.87$

$A = 221.74$

YOU TRY:

9.



$A = 25\pi$

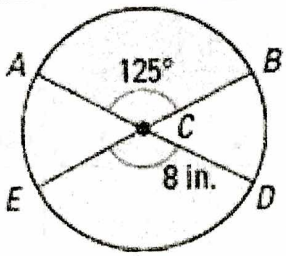
$\frac{160}{360} = \frac{x}{25\pi}$

$360x = 160 \cdot 25\pi$

$\frac{360x}{360} = \frac{12566.37}{360}$

$x = 34.91$

10.



$A = 64\pi$

$\frac{125}{360} = \frac{x}{64\pi}$

$360x = 125 \cdot 64\pi$

$\frac{360x}{360} = \frac{25132.74}{360}$

$x = 69.81$

$A = 139.62$

GEOMETRY NOTES

LESSON 49: Angle Measures in Polygons

CLASSIFYING POLYGONS	
Convex Polygon	diagonals are all inside the polygon
Concave Polygon	at least one diagonal is outside the figure
Equilateral	all sides congruent
Equiangular	all angles congruent
Regular	all sides and angles congruent
A Polygon is Not…	No curved sides No overlapping sides sides must all connect

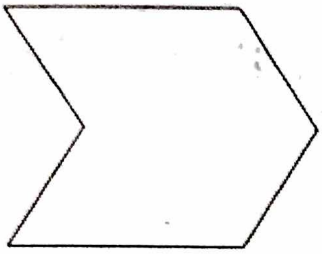
NAMING POLYGONS	
# OF SIDES	**POLYGON NAME**
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	septagon/heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

GEOMETRY NOTES

LESSON 49: Angle Measures in Polygons

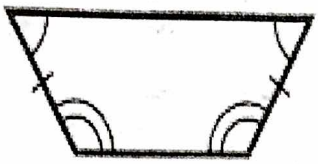
EXAMPLES: Determine whether the following are polygons. If yes, classify and name them. If no, explain.

1.



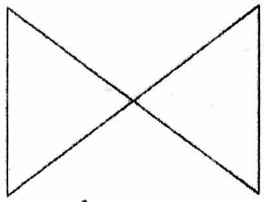
Yes - concave hexagon

2.



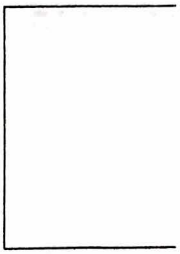
Yes - convex quadrilateral

3.



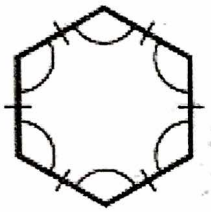
No - overlapping sides

4.



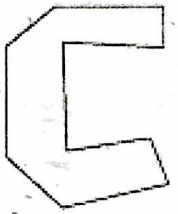
No - not closed

5.



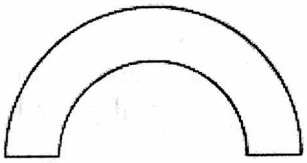
Yes - regular convex hexagon

6.



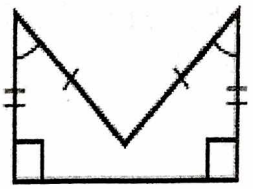
Yes - decagon

7.



No - curved sides

8.



Yes - concave pentagon

ANGLES IN A POLYGON:

EXPLORATION 1 Google the phrase "math open reference interior angles of polygons"	
Move around the points, change the number of sides and in the options change between regular and irregular	What do you notice about the angles on the inside of the polygons? Add to 360, 540, 720, 900, 1080, ... Each time you increase the sides of your polygon, how many degrees does the total go up by? 180
Make your polygon a regular polygon.	In a regular polygon, how do you get the angle measure for each individual angle? $\frac{\text{Total}}{\#\text{ of <s}}$

GEOMETRY NOTES

LESSON 49: Angle Measures in Polygons

INTERIOR ANGLE FORMULAS	**SUM OF INTERIOR ANGLES OF A POLYGON**	$(n-2)180$
	EACH INTERIOR ANGLE OF REGULAR POLYGON	$\frac{(n-2)180}{n}$

EXPLORATION 2

Google the phrase "math open reference exterior angles of polygons"

Move around the points, change the number of sides and in the options change between regular and irregular.	What do you notice about the angles on the outside of the polygons? Always add to 360 Each time you increase the sides of your polygon, how many degrees does the total go up by? 0
Make your polygon a regular polygon.	In a regular polygon, how do you get the angle measure for each exterior angle? $\frac{360}{n}$

EXTERIOR ANGLE FORMULAS	**SUM OF EXTERIOR ANGLES OF POLYGON**	360
	EACH EXTERIOR ANGLE OF REGULAR POLYGON	$\frac{360}{n}$

GEOMETRY NOTES

LESSON 49: Angle Measures in Polygons

EXAMPLES: Find the indicated measures.

9. Find the sum of the interior angles of a 13-gon.

$(13-2)180$

$11 \cdot 180 = \boxed{1980°}$

10. Find the sum of the exterior angles of a dodecagon.

$\boxed{360°}$

11. Find each interior angle of a regular octagon.

$(8-2)180$

$6 \cdot 180 = 1080$

$\frac{1080}{8} = \boxed{135°}$

12. Find each exterior angle of a regular pentagon.

$\frac{360}{5} = \boxed{72°}$

***13.** The measure of each interior angle of a regular polygon is 147.3°. How many sides does the polygon have?

$\frac{(n-2)180}{n} = 147.3$

$(n-2)180 = 147.3n$

$180n - 360 = 147.3n$

$-360 = -32.7n$

$\boxed{n = 11}$

***14.** The measure of each interior angle of a regular polygon is 165°. How many sides does the polygon have?

$\frac{(n-2)180}{n} = 165$

$(n-2)180 = 165n$

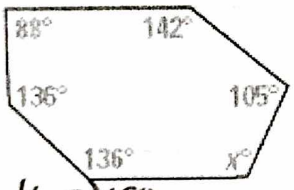
$180n - 360 = 165n$

$-360 = -15n$

$\boxed{n = 24}$

EXAMPLES: Find the value of x.

15.



$(6-2)180$

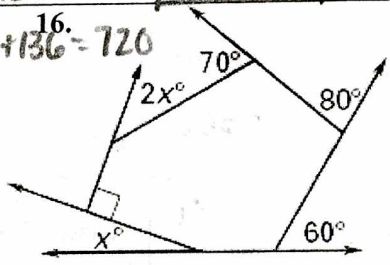
720

$88 + 142 + 105 + x + 136 + 136 = 720$

$x + 607 = 720$

$\boxed{x = 113}$

16.



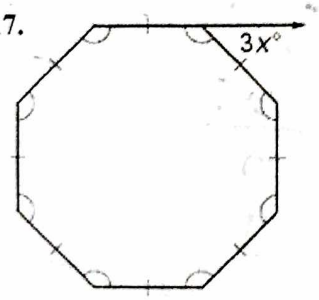
$x + 60 + 80 + 70 + 2x + 90 = 360$

$3x + 300 = 360$

$3x = 60$

$\boxed{x = 20}$

17.

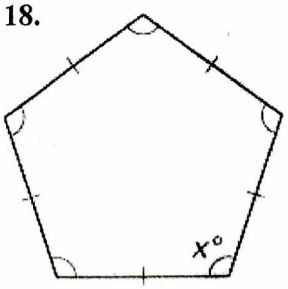


$\frac{360}{8} = 45$

$3x = 45$

$\boxed{x = 15}$

18.



$(5-2)180$

540

$5x = 540$

$\boxed{x = 108}$

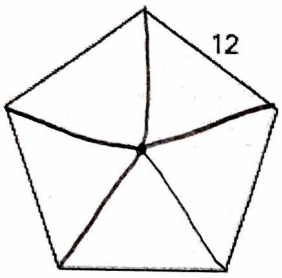
GEOMETRY NOTES

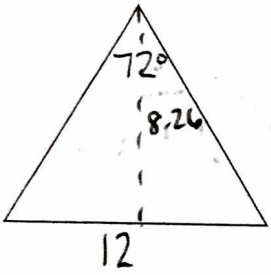
LESSON 50: Area of Regular Polygons

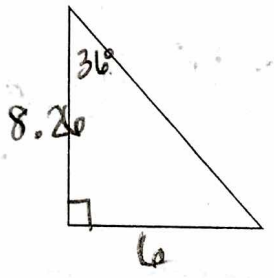
Regular Polygon- All sides + angles congruent

EXAMPLES: Find the area of the regular polygons shown.

1.

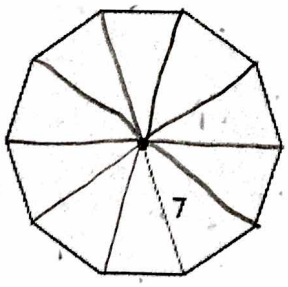


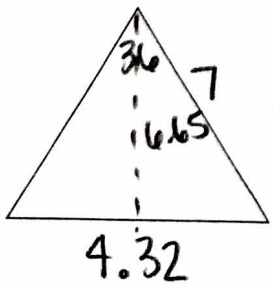


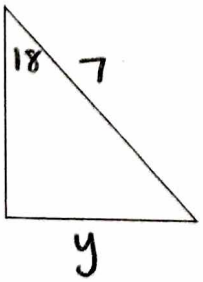


Central Angle:	$\frac{360}{5} = 72$
Apothem:	$\tan 36 = \frac{6}{x}$ $x = \frac{6}{\tan 36}$ $x = 8.26$
Side Length:	12
Area:	$A = \frac{1}{2} \cdot 12 \cdot 8.26$ $A = 5 \cdot 49.56$ $A = 49.56$ $\boxed{A = 247.8}$

2.







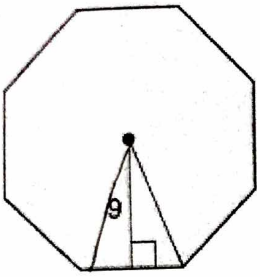
Central Angle:	$\frac{360}{10} = 36$
Apothem:	$\cos 18 = \frac{x}{7}$ $x = 7\cos 18$ $x = 6.65$
Side Length:	$\sin 18 = \frac{y}{7}$ $y = 7\sin 18$ $y = 2.16$
Area:	$A = \frac{1}{2} \cdot 6.65 \cdot 4.32$ $A = 10 \cdot 14.364$ $A = 14.364$ $\boxed{A = 143.64}$

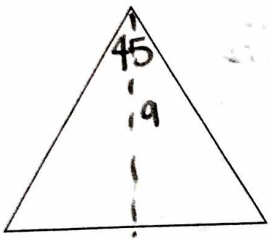
GEOMETRY NOTES

LESSON 50: Area of Regular Polygons

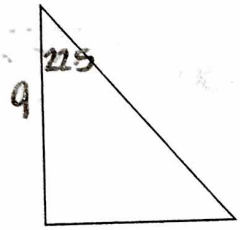
YOU TRY:

3.





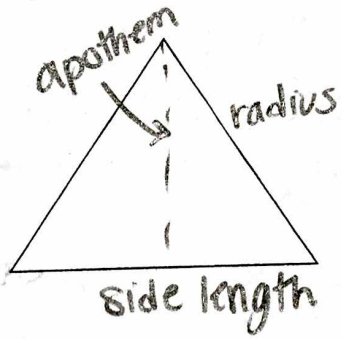
7.46



3.73

Central Angle:	$\frac{360}{8} = 45$
Apothem:	9
Side Length:	$\tan 22.5 = \frac{x}{9}$ $x = 9\tan 22.5$ $\quad x = 3.73$
Area:	$A = .5 \cdot 9 \cdot 7.46$ $\quad A = 8 \cdot 33.57$ $A = 33.57$ $\quad \boxed{A = 268.56}$

What if they don't give you a picture? How do you draw the figure? Here are some quick tips!



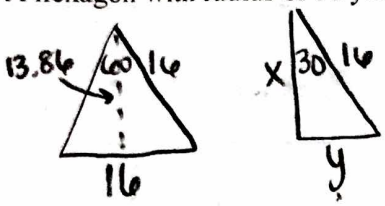
central <= $\frac{360}{\#\text{sides}}$

GEOMETRY NOTES

LESSON 50: Area of Regular Polygons

EXAMPLES: **Without drawing the polygon, draw the triangle you would use to calculate the area of the polygon.**

4. A hexagon with radius of 16 yd.



$\sin 30 = \frac{y}{16}$

$y = 16 \sin 30$

$y = 8$

$\cos 30 = \frac{x}{16}$

$x = 16 \cos 30$

$x = 13.86$

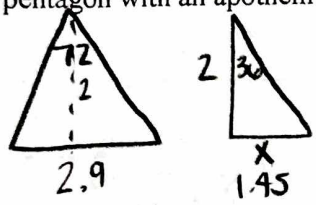
$A = .5 \cdot 16 \cdot 13.86$

$A = 110.88$

$A = 6 \cdot 110.88$

$A = 665.28 \text{ yd}^2$

5. A pentagon with an apothem of 2 ft.



$\tan 36 = \frac{x}{2}$

$x = 2 \tan 36$

$x = 1.45$

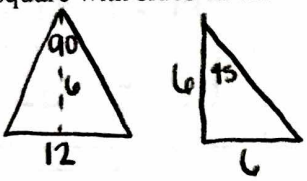
$A = .5 \cdot 2 \cdot 2.9$

$A = 2.9$

$A = 5 \cdot 2.9$

$A = 14.5 \text{ ft}^2$

6. A square with sides 12 in.



$A = .5 \cdot 6 \cdot 12$

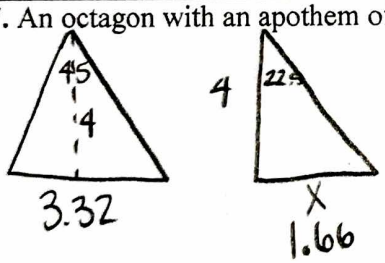
$A = 36$

$A = 4 \cdot 36$

$A = 144 \text{ in}^2$

YOU TRY:

7. An octagon with an apothem of 4 yd.



$\tan 22.5 = \frac{x}{4}$

$x = 4 \tan 22.5$

$x = 1.66$

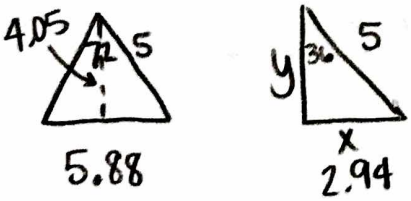
$A = .5 \cdot 4 \cdot 3.32$

$A = 6.64$

$A = 8 \cdot 6.64$

$A = 53.12 \text{ yd}^2$

8. A pentagon with a radius of 5 ft.



$\sin 36 = \frac{x}{5}$

$x = 5 \sin 36$

$x = 2.94$

$\cos 36 = \frac{y}{5}$

$y = 5 \cos 36$

$y = 4.05$

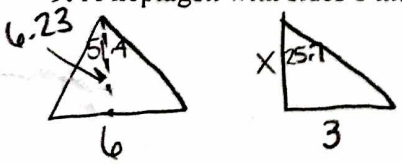
$A = .5 \cdot 5.88 \cdot 4.05$

$A = 11.907$

$A = 5 \cdot 11.907$

$A = 59.535 \text{ ft}^2$

9. A heptagon with sides 6 in.



$\tan 25.7 = \frac{3}{x}$

$x = \frac{3}{\tan 25.7}$

$x = 6.23$

$A = .5 \cdot 6 \cdot 6.23$

$A = 18.7$

$A = 7 \cdot 18.7$

$A = 130.9 \text{ in}^2$