GEOMETRY NOTES

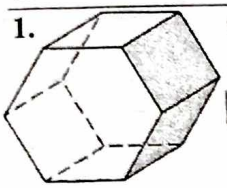
LESSON 51: Introduction to Three-Dimensional Figures

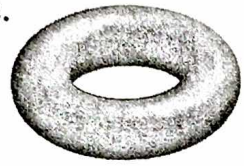
***POLYHEDRONS:** Closed 3D figures that is made up of flat polygon shaped regions

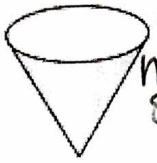
***PRISMS:** a solid with 2 bases that are parallel

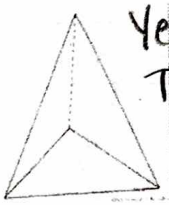
***PYRAMIDS:** a solid with only 1 base, all other lateral faces meet at 1 vertex

EXAMPLES: Tell whether the solid is a polyhedron. If it is, name the polyhedron.

1.  Yes Hexagonal Prism

2.  No- not flat surfaces

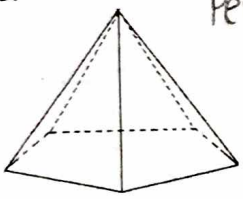
3.  No- not flat surfaces

4.  Yes Triangular Pyramid

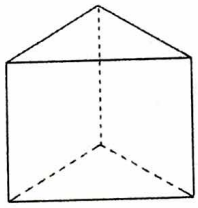
***FACES:** flat regions ***EDGES:** segment where 2 faces intersect ***VERTICES:** point where 3 or more edges intersect

EULER'S FORMULA: $F+V-E=2$

EXAMPLES: Name the polyhedron. Then find the number of faces, edges, and vertices in each polyhedron.

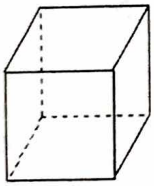
5. 

Pentagonal Pyramid
F: 6
E: 10
V: 6
$F+V-E=2$
$6+6-10=2$ ✓

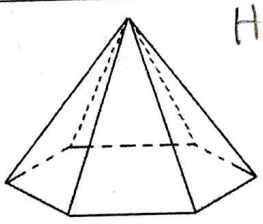
6. 

Triangular Prism
F: 5
E: 9
V: 6
$F+V-E=2$
$5+6-9=2$ ✓

YOU TRY:

7. 

Quadrilateral Prism
F: 6
E: 12
V: 8
$6+8-12=2$ ✓

8. 

Hexagonal Pyramid
F: 7
E: 12
V: 7
$7+7-12=2$ ✓

EXAMPLES:

9. Use Euler's Formula to calculate how many edges a polyhedron has if it has 6 faces and 7 vertices.

$F+V-E=2$
$6+7-E=2$
$13-E=2$ **E=11**

10. Use Euler's Formula to calculate how many vertices a polyhedron has if it has 9 faces and 21 edges.

$F+V-E=2$
$9+V-21=2$
$V-12=2$ **V=14**

YOU TRY:

11. Use Euler's Formula to calculate how many vertices a polyhedron has if it has 12 faces and 30 edges.

$F+V-E=2$
$12+V-30=2$
$V-18=2$ **V=20**

12. Use Euler's Formula to calculate how many faces a polyhedron has if it has 6 edges and 8 vertices.

$F+V-E=2$
$F+8-6=2$
$F+2=2$ **F=0** No polyhedron possible

GEOMETRY NOTES

LESSON 52: Surface Area of Prisms and Cylinders

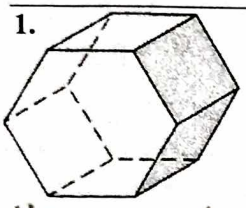
Prism: Polyhedron w/ two polygon bases (that are ≅)

SURFACE AREA OF PRISMS: 2(area of base) + area of lateral faces

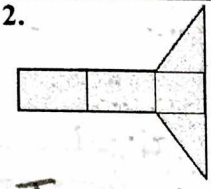
Cylinder: Two ≅ circle bases connected by a rectangle

SURFACE AREA OF CYLINDERS: $2\pi r^2 + 2\pi rh$

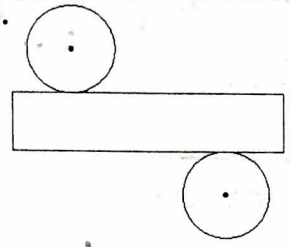
EXAMPLES: Name each solid.

1. 

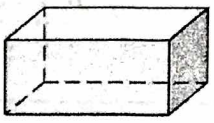
Hexagonal Prism

2. 

Triangular Prism

3. 

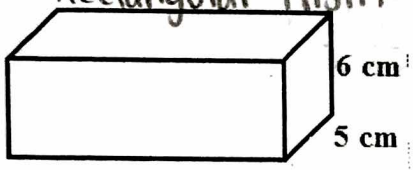
Cylinder

4. 

Rectangular Prism

EXAMPLES: Name the solid. Then find its surface area. Round to nearest tenth.

5. Rectangular Prism



F+B: 6, 11 — $A = 6 \cdot 11$, $A = 66$

Side: 6, 5 — $A = 6 \cdot 5$, $A = 30$

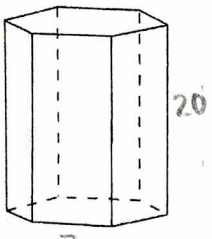
T+B: 5, 11 — $A = 11 \cdot 5$, $A = 55$

Total: $2 \cdot 66 + 2 \cdot 30 + 2 \cdot 55$

$= 132 + 60 + 110$

$= \boxed{302\ cm^2}$

6.



Side Lengths: 7 cm
Height: 20 cm

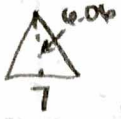
Base: 7, 7, 30°, 3.5

$\tan 30 = \frac{3.5}{x}$

$x \cdot \tan 30 = 3.5$

$x = \frac{3.5}{\tan 30}$

$x = 6.06$



$A = \frac{1}{2} \cdot 7 \cdot 6.06$

$A = 21.21$

$A = 6 \cdot 21.21$

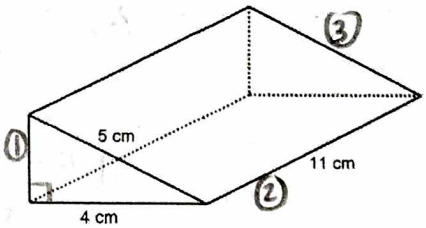
$A = 127.26$

LF: 7, 20 — $A = 7 \cdot 20$, $A = 140$

Total: $2(127.26) + 6(140)$

$= \boxed{1094.52\ cm^2}$

7.



Base: 5, 4, $x = 3$ — $A = \frac{1}{2} \cdot 3 \cdot 4$, $A = 6$

LF1: 3, 11 — $A = 3 \cdot 11$, $A = 33$

LF2: 4, 11 — $A = 4 \cdot 11$, $A = 44$

LF3: 5, 11 — $A = 5 \cdot 11$, $A = 55$

Total: $2(6) + 33 + 44 + 55$

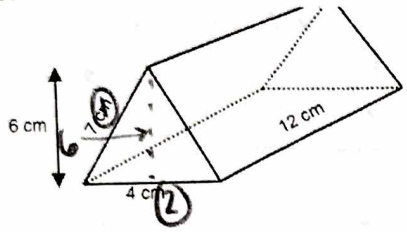
$= \boxed{144\ cm^2}$

GEOMETRY NOTES

LESSON 52: Surface Area of Prisms and Cylinders

YOU TRY:

8.



Base: $A = \frac{1}{2} \cdot 4 \cdot 6$, $A = 12$

LF1: $A = 7 \cdot 12$, $A = 84$

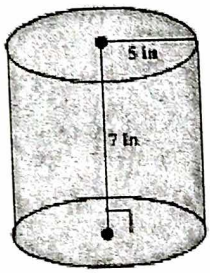
LF2: $A = 4 \cdot 12$, $A = 48$

Total: $2(12) + 2(84) + 48$

$= 240 cm^2$

EXAMPLES: Find the surface area of the cylinder. Round to nearest tenth.

9.



Base: πr^2

$= \pi 5^2$

$= 25\pi$

LF: $2\pi rh$

$2\pi 5 \cdot 7$

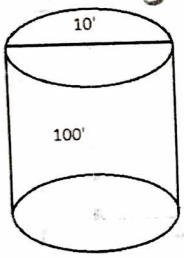
70π

Total $= 25\pi + 25\pi + 70\pi$

$= 377 in^2$

YOU TRY:

10. $r = 5$



Base: πr^2

$= \pi 5^2$

$= 25\pi$

LF: $2\pi rh$

$= 2\pi \cdot 5 \cdot 100$

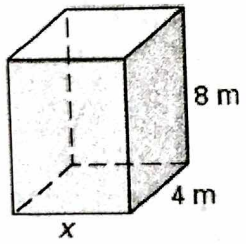
$= 1000\pi$

Total $= 1000\pi + 25\pi + 25\pi$

$= 3298.67 ft^2$

EXAMPLES: Solve for the variable given the surface area S of the prism and cylinder.

11. $S = 208 m^2$



F+B: $A = 8x$

T+B: $A = 4x$

Sides: $A = 32$

$2(8x) + 2(4x) + 2(32) = 208$

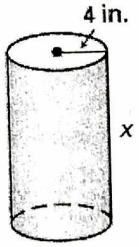
$16x + 8x + 64 = 208$

$24x + 64 = 208$

$24x = 144$

$x = 6$

12. $S = 452.4 in.^2$



Base: $A = \pi r^2$

$A = \pi 4^2$

$A = 16\pi$

LF: $2\pi rh$

$2\pi 4 \cdot x$

$452.4 = 2(16\pi) + 8\pi x$

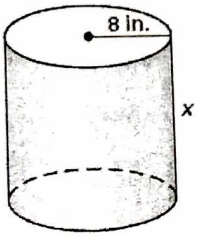
$452.4 = 32\pi + 8\pi x$

$\frac{351.87}{8\pi} = \frac{8\pi x}{8\pi}$

$x = 14$

YOU TRY:

13. $S = 1206.4 in.^2$



Base: $A = \pi 8^2$

$A = 64\pi$

LF: $A = 2\pi rh$

$= 2\pi \cdot 8 \cdot x$

$= 16\pi x$

$1206.4 = 2(64\pi) + 16\pi x$

$1206.4 = 128\pi + 16\pi x$

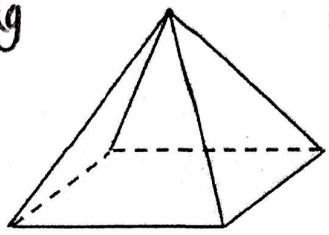
$804.28 = 16\pi x$

$x = 16$

GEOMETRY NOTES

LESSON 53: Surface Area of Pyramids and Cones

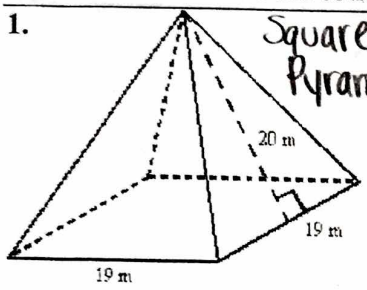
Pyramid: polygon base with all lateral faces meeting at 1 point



SURFACE AREA OF PYRAMID: SA = area of base + area of triangles

EXAMPLES: Name the solid. Then find its surface area. Round to nearest tenth.

1.



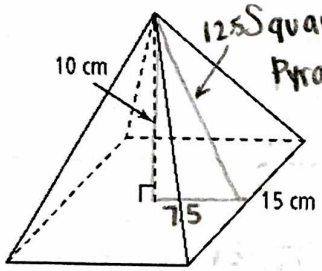
Square Pyramid

Base: $A = 19 \cdot 19$, $A = 361$

LF: $A = \frac{1}{2} \cdot 19 \cdot 20$, $A = 190$

Total $= 361 + 4(190) = 1121\ m^2$

2.



$10^2 + 7.5^2 = x^2$

$100 + 56.25 = x^2$

$x = 12.5$

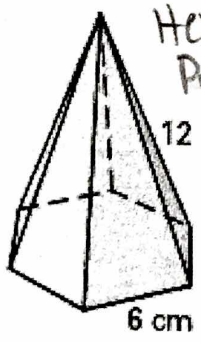
Base: $A = 15 \cdot 15$, $A = 225$

LF: $A = \frac{1}{2} \cdot 12.5 \cdot 15$, $A = 93.75$

Total $= 225 + 4(93.75)$

Total $= 600\ cm^2$

3.



Hexagonal Pyramid

Base: $\tan 30 = \frac{3}{x}$

$x \tan 30 = 3$

$x = \frac{3}{\tan 30}$

$x = 5.2$

$A = \frac{1}{2} \cdot 6 \cdot 5.2$

$A = 15.6$

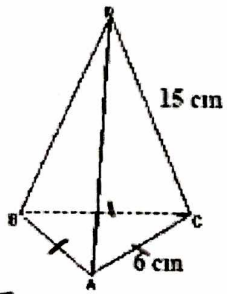
$A = 93.6$

LF: $A = \frac{1}{2} \cdot 6 \cdot 12$, $A = 36$

Total $= 93.6 + 6(36) = 309.6\ cm^2$

YOU TRY:

4.



Triangular Prism

Base: $A = \frac{1}{2} \cdot 6 \cdot 1.73$

$= 5.19$

$A = 3 \cdot 5.19$

$A = 15.57$

LF: $A = \frac{1}{2} \cdot 6 \cdot 15$

$= 45$

Total $= 15.57 + 3(45) = 150.57\ cm^2$

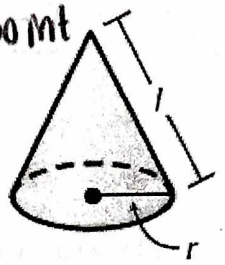
$\tan 60 = \frac{3}{x}$

$x = \frac{3}{\tan 60} = 1.73$

GEOMETRY NOTES

LESSON 53: Surface Area of Pyramids and Cones

Cone: circle base with curved face that meets at a point



SURFACE OF A CONE:

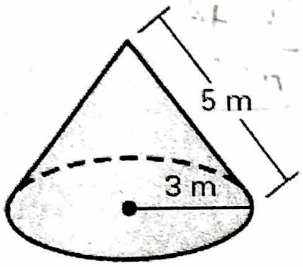
$A = \pi r l + \pi r^2$

r = radius

l = slant height

EXAMPLES: Find the surface area of the cone.

5.



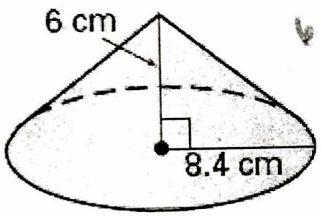
$A = \pi \cdot 3 \cdot 5 + \pi 3^2$

$A = 15\pi + 9\pi$

$A = 24\pi$

$\boxed{A = 75.4\ m^2}$

6.



$6^2 + 8.4^2 = x^2$

$36 + 70.56 = x^2$

$x^2 = 106.56$

$x = 10.32$

$A = \pi \cdot 8.4 \cdot 10.32 + \pi\, 8.4^2$

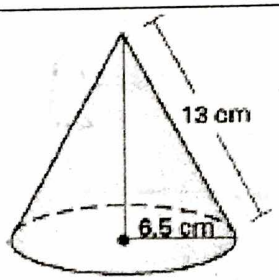
$A = 86.69\pi + 70.56\pi$

$A = 157.25\pi$

$\boxed{A = 494.02\ cm^2}$

YOU TRY:

7.



$A = \pi \cdot 6.5 \cdot 13 + \pi 6.5^2$

$A = 84.5\pi + 42.25\pi$

$A = 126.75\pi$

$\boxed{A = 398.2\ cm^2}$

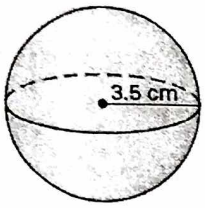
GEOMETRY NOTES

LESSON 54: Surface Area of Spheres and Stacked Solids

	SPHERE	HEMI-SPHERE
SURFACE AREA:	$S = 4\pi r^2$	$SA = 2\pi r^2 + \pi r^2 = 3\pi r^2$ (w/ base) $SA = 2\pi r^2$ (w/out base)

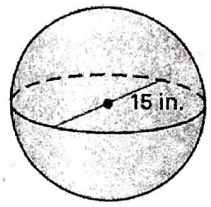
EXAMPLES: Find the surface area of each sphere.

1.



$A = 4\pi 3.5^2$
$A = 4\pi \cdot 12.25$
$A = 49\pi$
$A = 153.94 \text{ cm}^2$

2. **YOU TRY**

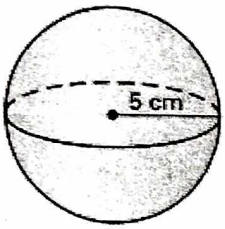


$r = 7.5$

$A = 4\pi \cdot 7.5^2$
$A = 4\pi \cdot 56.25$
$A = 225\pi$
$A = 706.86 \text{ in}^2$

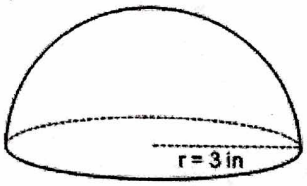
EXAMPLES: Find the surface area of each hemisphere.

3.



$A = 2\pi 5^2$
$A = 2\pi \cdot 25$
$A = 50\pi$
$A = 157.08 \text{ cm}^2$

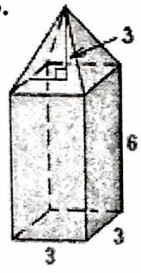
4.



$A = 3\pi 3^2$
$A = 3\pi \cdot 9$
$A = 27\pi$
$A = 84.82 \text{ in}^2$

Find the surface area of the solid.

5.



Pyramid: $3^2 + 1.5^2 = x^2$
$x = 3.35$
$A = \frac{1}{2} \cdot 3 \cdot 3.35$
$A = 5.025$
$A = 4 \cdot 5.025 = 20.1$

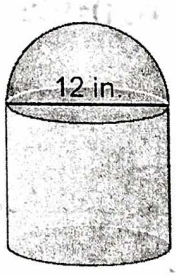
Prism: Base: $\square 3$ $A = 9$

LF: $\square 6$ $A = 18$

$A = 9 + 4(18) = 81$

Total: 100.1

6.



Hemisphere: $2\pi r^2$
$2\pi 6^2$
$36 \cdot 2\pi$
72π

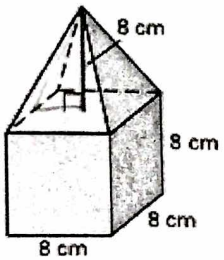
Cylinder: Base: $A = \pi r^2$
$= 6^2\pi$
$= 36\pi$

LF: $2\pi rh$
$2\pi \cdot 6 \cdot 13$
156π

Total: 264π
$= 829.38 \text{ in}^2$

YOU TRY:

7.



Pyramid: $A = \frac{1}{2} \cdot 8 \cdot 8.94$
$A = 35.76$
$A = 4 \cdot 35.76$
$A = 143.04$

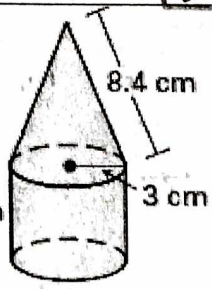
$x = 8.94$

Prism: B: $\square 8$ $A = 64$

LF: $\square 8$ $A = 64$

Total: $143.04 + 64 + 4(64)$
$= 463.04 \text{ cm}^2$

8.



Cone: $\pi r l$
$= \pi \cdot 3 \cdot 8.4$
$= 25.2\pi$
$= 79.17 \text{ cm}^2$

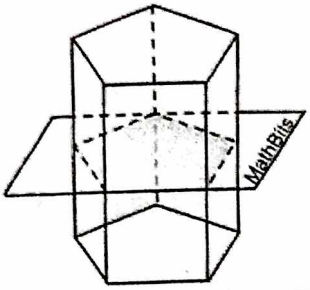
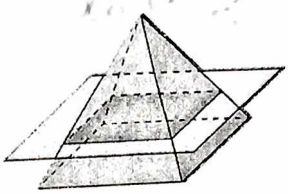
Cylinder: B: $\pi 3^2$
$= 9\pi$
$= 28.27$

LF: $2\pi rh$
$= 2\pi \cdot 3 \cdot 5.1$
$= 30.6\pi$
$= 96.13$

Total: 203.57 cm^2

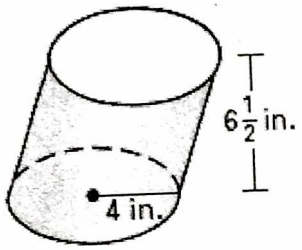
GEOMETRY NOTES

LESSON 55: Volume of Prisms/Cylinders and Pyramids/Cones

VOLUME OF PRISMS/CYLINDERS	VOLUME OF PYRAMIDS/CONES
 $V = B \cdot h$ B = area of base	 $V = \frac{1}{3}Bh$ B = area of base

EXAMPLES: Name each solid. Then find the volume of each solid.

1.



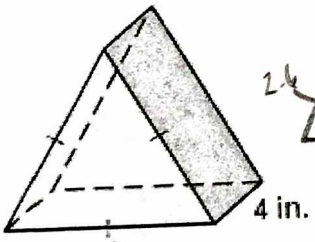
cylinder

$B = \pi r^2$
$= \pi 4^2$
$= 16\pi$

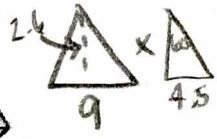
$V = 16\pi \cdot 6.5$

$V = 326.73 \text{ in}^3$

2.



prism



$B = \frac{1}{2} \cdot 9 \cdot 2.6$

$B = 11.7$

$B = 35.1$

$\tan 60 = \frac{4.5}{x}$

$x \tan 60 = 4.5$

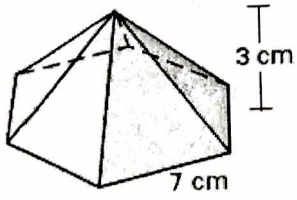
$x = \frac{4.5}{\tan 60}$

$x = 2.6$

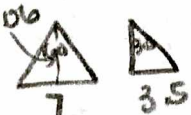
$V = 35.1 \cdot 4$

$V = 140.4 \text{ in}^3$

3.



pyramid

Base: 

$\tan 30 = \frac{3.5}{x}$

$x \tan 30 = 3.5$

$x = \frac{3.5}{\tan 30}$

$x = 6.06$

$A = \frac{1}{2} \cdot 7 \cdot 6.06$

$A = 21.21$

$A = 6 \cdot 21.21 = 127.26$

$V = \frac{1}{3} \cdot 127.26 \cdot 3$

$V = 127.26 \text{ cm}^3$

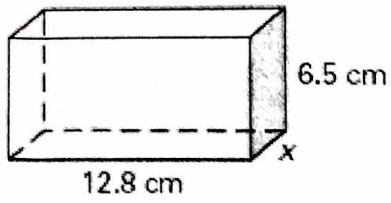
GEOMETRY NOTES

LESSON 55: Volume of Prisms/Cylinders and Pyramids/Cones

EXAMPLES: Solve for the variable using the given measurements.

4.

Volume = 200 cm³



prism

$B = 12.8x$

$V = Bh$

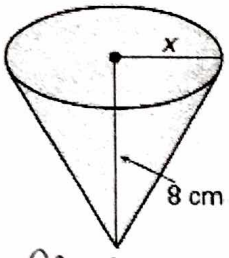
$200 = 12.8x \cdot 6.5$

$\frac{200}{83.2} = \frac{83.2x}{83.2}$

$x = 2.4$ cm

5.

Volume = 170 cm³



Cone

$B = \pi x^2$

$V = \frac{1}{3} \cdot \pi x^2 \cdot 8$

$\frac{170}{8.38} = \frac{8.38x^2}{8.38}$

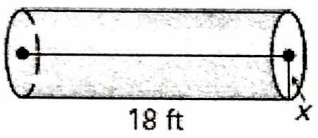
$x^2 = 20.29$

$x = 4.5$ cm

YOU TRY

6.

. Volume = 475 ft³



Cylinder

$B = \pi x^2$

$V = Bh$

$475 = \pi x^2 \cdot 18$

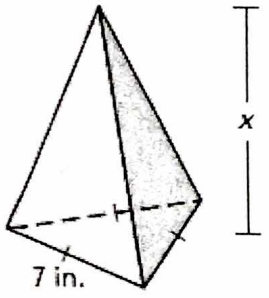
$\frac{475}{18\pi} = \frac{18\pi x^2}{18\pi}$

$x^2 = 8.4$

$x = 2.9$ ft

7.

Volume = 81 in.³



B: $\frac{1}{2} \cdot 7 \cdot x$ (60°, 3.5)

$\tan 60 = \frac{3.5}{x}$

$x \tan 60 = 3.5$

$x = \frac{3.5}{\tan 60}$

$x = 2.02$

$A = \frac{1}{2} \cdot 7 \cdot 2.02$

$A = 7.07$

$A = 21.21$

$V = \frac{1}{3} Bh$

$81 = \frac{1}{3} \cdot 21.21 h$

$\frac{81}{7.07} = \frac{7.07h}{7.07}$

$h = 11.46$ in

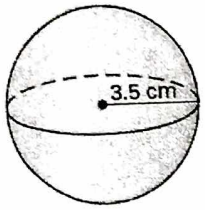
GEOMETRY NOTES

LESSON 56: Volume of Spheres and Stacked Solids

	SPHERE	HEMI-SPHERE
VOLUME:	$V=\frac{4}{3}\pi r^3$	$V=\frac{2}{3}\pi r^3$

EXAMPLES: Find the volume of each sphere.

1.



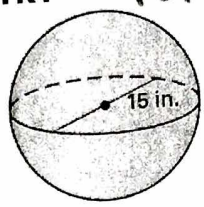
$V=\frac{4}{3}\pi 3.5^3$

$V=\frac{4}{3}\pi \cdot 42.875$

$V=57.16\pi$

$V=179.59\ cm^3$

2. **YOU TRY** $r=7.5$ $V=\frac{4}{3}\pi 7.5^3$

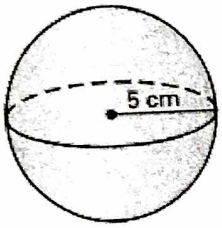


$V=\frac{4}{3}\pi \cdot 421.875$

$V=1767.15\ in^3$

EXAMPLES: Find the volume of each hemisphere.

3.

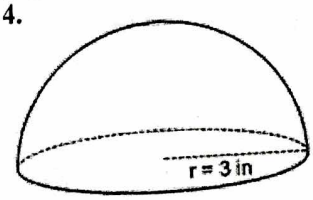


$V=\frac{2}{3}\pi 5^3$

$V=\frac{2}{3}\pi \cdot 125$

$V=261.8\ cm^3$

4.



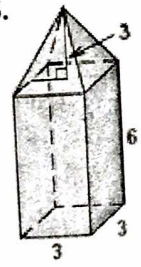
$V=\frac{2}{3}\pi \cdot 3^3$

$V=\frac{2}{3}\pi \cdot 27$

$V=56.55\ in^3$

Find the volume of the solid.

5.



Pyramid: $B=3\cdot 3=9$

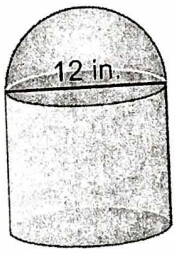
$V=\frac{1}{3}\cdot 9\cdot 3=9$

Prism: $B=3\cdot 3=9$

$V=9\cdot 6=54$

$V=63$

6.



Hemisphere: $V=\frac{2}{3}\pi 6^3$

$V=452.39\ in^3$

Cylinder $B=\pi 6^2$

$=36\pi$

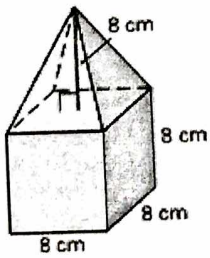
$V=36\pi \cdot 13$

$V=1470.27$

$V=1922.66\ in^3$

YOU TRY:

7.



Pyramid: $\square$ 8 $A=64$

$V=\frac{1}{3}\cdot 64\cdot 8$

$V=170.67$

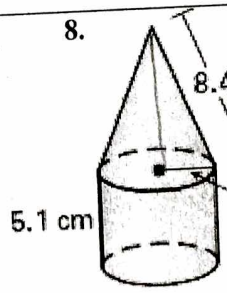
Prism: $B=8\cdot 8=64$

$V=64\cdot 8$

$V=512$

Total: $682.67\ cm^3$

8.



Cone: $V=\frac{1}{3}\cdot 9\pi \cdot 7.85$

$V=73.98$

Cylinder: $V=9\pi \cdot 5.1$

$V=144.2$

$V=218.18\ cm^3$

$B=\pi 3^2$

$=9\pi$

7.85 8.4 3