Lesson #53
Introduction to Solving Quadratics

Success Criteria: I can match different forms of the same polynomial together.

Review: FACTORS are numbers or expressions that are MULTIPLIED together.

So in the expression $5(x+2)(x-4)$, there are three FACTORS.

The factors are 5 and $x+2$ and $x-4$.

Problem #1:

Each equation in the first column is equivalent to one of the equations in the second column and is also equivalent to one of the equations in the third column. Use previously learned rewriting skills to figure out which ones match, then write the correct matches in the three boxes provided below.

Factored Form	Standard Form	Nonstandard Form
a. $(x+1)(x-3)=0$ $X^2-2X-3=0$	A. $x^2-4x+3=0$	1. $x^2+2x=3$
b. $(x-1)(x+3)=0$ $X^2+2X-3=0$	B. $x^2-2x-3=0$	2. $x^2-4x=-3$
c. $(x-1)(x-3)=0$ X^2-4X+3	C. $x^2+2x-3=0$	3. $x^2-2x=3$

Matching Set	Matching Set	Matching Set
a B 3	b C 1	c A 2

Success Criteria: I can identify values of x that will make a polynomial equal zero.

Problem #2:

(A) Find the value of the product $3(x+2)(x-4)$ for each given x-value by substituting.

Example: When $x = 5$, the product is $3(5+2)(5-4) = 3(7)(1) = 21$

When $x = 1$, the product is? $3(1+2)(1-4) = -27$

When $x = 0$, the product is? $3(0+2)(0-4) = -24$

When $x = 4$, the product is? $3(4+2)(4-4) = 0$

When $x = -2$, the product is? $3(-2+2)(-2-4) = 0$

When $x = 2$, the product is? $3(2+2)(2-4) = -24$

(B) Based on your observations of the above results, complete the following statement:

" *Any time ONE of the factors is* 0, *the entire product will be* 0 ."

(C) Based on your observations of the above results, find all values of x that will result in a value of 0 for the product $-6(x-5)(x+4)$.

$x-5=0$ $\quad$ $x+4=0$

$x=5$ $\quad$ $x=-4$

Lesson #54
Solving Polynomial Equations in Factored Form

Success Criteria: I can solve polynomials in factored form by setting all parts equal to zero. I can identify the x values that are zeros on a table, graph and in the equation.

A polynomial is in ***Factored Form*** if it is written as the **product of 2 or more linear factors.**

$(2x + 3)(x - 5) = 0$ **Factored Form**

$2x^2 - 7x - 15 = 0$ Standard form

A value of x that makes any of the factors zero is a solution of the polynomial equation. It is also called a zero, a root, and an x-intercept.

Example #1:

Solve the equation $(x-2)(x+1)=0$:

a. Set each factor = 0.

$x-2=0$ $\quad$ $x+1=0$

$x=2$ $\quad$ $x=-1$

b. What values of x make the answer to each factor 0?

2, -1

These x's make the entire equation equal 0. This is the **Zero-Product Property**.

c. Check your solutions by substituting them into the equation.

Check $x=2$:	Check $x=-1$:
$(x-2)(x+1)=0$	$(x-2)(x+1)=0$
$(2-2)(2+1)=0$	$(-1-2)(-1+1)=0$
$0\cdot 3=0$	$-3\cdot 0=0$
$0=0$ ✓	$0=0$ ✓

Zero-Product Property:

Words: If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra: If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

Example #2: Find the solutions to each polynomial equation.

1. $(x+4)(x-1)=0$

$x+4=0$ $\quad$ $x-1=0$

$x=-4$ $\quad$ $x=1$

$\boxed{x=-4,1}$

2. $(7x+2)(6x-3)=0$

$7x+2=0$ $\quad$ $6x-3=0$

$7x=-2$ $\quad$ $6x=3$

$x=\frac{-2}{7}$ $\quad$ $x=\frac{1}{2}$

$\boxed{x=\frac{1}{2},-\frac{2}{7}}$

3. $x(5x-2)=0$

$x=0$ $\quad$ $5x-2=0$

$5x=2$

$x=\frac{2}{5}$

$\boxed{x=0,\frac{2}{5}}$

4. $5(x+11)(3x-4)=0$

$x+11=0$ $\quad$ $3x-4=0$

$x=-11$ $\quad$ $3x=4$

$x=\frac{4}{3}$

$\boxed{x=\frac{4}{3},-11}$

Example #3:

a) Is $x = 7$ a solution to the equation $3x(x-7)(9x-1) = 0$? Why or why not?

Yes, when $X-7=0$ $X=7$

b) Is $x = 6$ a solution to the equation $(x-4)(x+6)(4x+3) = 0$? Why or why not?

No because $X-4=0$, $X+6=0$, and $4X+3=0$ does not give you $X=6$

A <u>solution</u> of the polynomial equation is also called a <u>zero</u>, a <u>root</u>, and an <u>x-intercept</u>.
This means it is the x-value where the graph crosses the x-axis.

Example #4:

$y = (x-3)(x+4)$

a) Where do you think it will cross the x-axis?

3, -4

b) Graph $y_1 = (x-3)(x+4)$ on the calculator.

c) Look at the graph. What do you notice?

Crosses at 3, -4

d) Look at the table. What do you notice?

(3,0) (-4,0)

Lesson #55
Factoring Polynomials Using the GCF

Success Criteria: I can find the GCF between two sets of numbers.

Prime/Composite Game

Review: Find the Greatest Common Factor for each pair. (Make a factor tree if necessary)

a. 9, 18

9

b. 42, 7

7

c. 15, 12

3

d. 32, 20

4

e. x^2, x^5

x^2

f. $4x^7, 6x^3$

$2x^3$

First Rule of Factoring: Take out the GCF

Success Criteria: I can find the GCF between two or more terms. I can factor out a GCF and solve a polynomial.

Example #1: Factor using the Greatest Common Factor (GCF)

Hint: Go "piece by piece" and find what they have in common. Always remove a "– " in front.

Factor:	Check:
1. $4x^2 + 6x$ $2x(2x+3)$ $\cancel{2}\cdot 2\cdot \cancel{x}\cdot x$ $\cancel{2}\cdot 3\cdot \cancel{x}$	$2x(2x+3)$ $4x^2+6x$ ✓
2. $-4r^2 - 12r$ $-4r(r+3)$ $-\cancel{2}\cdot 2\cdot \cancel{r}\cdot r$ $-\cancel{2}\cdot 6\cdot \cancel{r}$	$-4r(r+3)$ $-4r^2-12r$ ✓
3. $4x^4 + 6x^3 + 2x^2$ $2x^2(2x^2+3x+1)$ $\cancel{2}\cdot 2\cdot \cancel{x}\cdot \cancel{x}\cdot x\cdot x$ $\cancel{2}\cdot 3\cdot \cancel{x}\cdot \cancel{x}\cdot x$ $\cancel{2}\cdot \cancel{x}\cdot \cancel{x}$	$2x^2(2x^2+3x+1)$ $4x^4+6x^3+2x^2$ ✓

Example#2: Using Factoring to Solve

1. Make sure the equation is set = 0.
2. Factor using the GCF.
3. Use the Zero-Product Property to find the solutions.

4. $4x^2 + 24x = 0$

$4x(x+6)=0$

$4x=0$ $x+6=0$

$x=0$ $x=-6$

$\boxed{x=0,-6}$

5. $8x^2 = 16x$

$8x^2-16x=0$

$8x(x-2)=0$

$8x=0$ $x-2=0$

$x=0$ $x=2$

$\boxed{x=0,2}$

6. $6x^2 = 15x$

$6x^2-15x=0$

$3x(2x-5)=0$

$3x=0$ $2x-5=0$

$x=0$ $2x=5$

$x=\frac{5}{2}$

$\boxed{x=0,\frac{5}{2}}$

7. $2x^3 - 16x^2 = 0$

$2x^2(x-8)=0$

$2x^2=0$ $x-8=0$

$x^2=0$ $x=8$

$x=0$

$\boxed{x=0,8}$

Lesson #56
Factoring $x^2 + bx + c$

Success Criteria: I can factor polynomials. I can solve a polynomial in factored form.

Warm-Up:

Factor using the Greatest Common Factor and the Solve.

1. $16x^2 + 28x = 0$

$4x(4x+7)=0$

$4x=0 \qquad 4x+7=0$

$x=0 \qquad 4x=-7$

$x=-\frac{7}{4}$

$\boxed{x=0,-\frac{7}{4}}$

$\cancel{4}\cdot\cancel{4}\cdot 2\cdot 2\cdot \cancel{x}\cdot x$

$\cancel{4}\cdot\cancel{4}\cdot 7\cdot \cancel{x}$

Multiplying/Simplifying:

Starts with $(x+5)(x-9)$

Foils to $x^2 - 9x + 5x - 45$

And becomes $x^2 - 4x - 45$

Factoring:

Starts with $x^2 - 4x - 45$

And becomes $(x+5)(x-9)$

Factoring $x^2 + bx + c$

Find the factors of c that add together to give b.

(For today, a = 1)

c

b

*Remember to always keep the first rule of factoring in mind: take out the GCF

Examples: Factor each expression

1. $n^2 - 6n + 8$

$\boxed{(n-4)(n-2)}$

8

-4 -2

-6

$n^2-4n-2n+8$

n^2-6n+8

2. $y^2 + 2y - 15$ (Check answer by distributing)

-15

-3 5

2

$\boxed{(y+5)(y-3)}$

$y^2+5y-3y-15$

$y^2+2y-15$

3. $-16 + 6w + w^2$

$w^2+6w-16$

-16

-2 8

6

$\boxed{(w-2)(w+8)}$

$w^2-2w+8w-16$

$w^2+6w-16$

4. $b^2 - 10b + 25$

$\boxed{(b-5)(b-5)}$

$b^2-5b-5b+25$

$b^2-10b+25$

Examples: Solve each equation.

5. $x^2 \cancel{-4x} + 24 = 0$ $-2x$

$x^2 - 2x + 24 = 0$

$(x+4)(x-6) = 0$

$x+4=0$ $\quad$ $x-6=0$

$x=-4$ $\quad$ $x=6$ $\quad$ $\boxed{x=-4,6}$

6. $m^2 - m = 42$

$m^2 - m - 42 = 0$

$(m+6)(m-7) = 0$

$m+6=0$ $\quad$ $m-7=0$

$m=-6$ $\quad$ $m=7$ $\quad$ $\boxed{m=-6,7}$

Please remember to always keep the first rule of factoring in mind: take out the GCF

EXAMPLES: Factor each expression

1. $5b^2 - 50b + 125$

$5(b^2 - 10b + 25)$

$5(b-5)(b-5)$

2. $14x^5 + 12x^3$

$2x^3(7x^2 + 6)$

Lesson #57
Factoring $x^2 + bx + c$ (continued)

Success Criteria: I can move all terms to one side of an equation and solve by factoring. I can use factoring to solve a real world situation.

Review: Factor each expression. Remember to always check for a GCF **!**

1. $x^2 - 18x + 77$

$\boxed{(x-11)(x-7)}$

$x^2 - 7x - 11x + 77$

$x^2 - 18x + 77$

2. $r^2 + 12r - 28$ (Check answer by distributing)

$\boxed{(r+14)(r-2)}$

$r^2 - 2r + 14r - 28$

$r^2 + 12r - 28$

Solving Quadratic Equations by Factoring

1. Set quadratic equation equal to zero. **Keep x^2 positive**
2. Factor.
3. Use the zero-product property.
4. Solve.

before you factor, make sure your equation is in standard form: $x^2 + bx + c = 0$

EXAMPLES: Solve

3. $x^2 + 9x = -14$

$x^2 + 9x + 14 = 0$

$(x+2)(x+7) = 0$

$x+2=0 \quad x+7=0$

$x=-2 \quad x=-7$

$\boxed{x=-2,-7}$

4. $18 + 3x = x^2$

$0 = x^2 - 3x - 18$

$0 = (x+3)(x-6)$

$x+3=0 \quad x-6=0$

$x=-3 \quad x=6$

$\boxed{x=-3,6}$

5. $-x^2 + 10 = 3x$

$0 = x^2 + 3x - 10$

$0 = (x+5)(x-2)$

$x+5=0 \quad x-2=0$

$x=-5 \quad x=2$

$\boxed{x=2,-5}$

6. $x^3 - 11x^2 + 18x = 0$

$x(x^2 - 11x + 18) = 0$

$x(x-9)(x-2) = 0$

$x=0 \quad x-9=0 \quad x-2=0$

$x=9 \quad x=2$

$\boxed{x=0,2,9}$

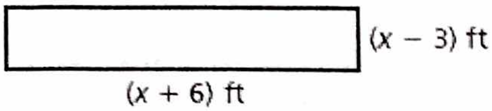
*Be careful that you read the directions when doing your homework and taking a quiz/test. Sometimes you are asked to solve and sometimes you are asked to simply factor.

Applications

7. Find the dimensions of the polygon with the given area.

Area $=$ 22 square feet

$A = l \cdot w$



$22 = (x+6)(x-3)$

$22 = x^2 - 3x + 6x - 18$

$0 = x^2 + 3x - 40$

$0 = (x+8)(x-5)$

$x+8=0 \quad x-5=0$

$x=-8 \quad x=5$

$\boxed{x=-8,5}$

8.

A wooden door contains a rectangular window. The width of the window is $(x - 2)$ feet. The area of the window can be represented by $x^2 - 3x + 2$.

$x^2-3x+2 = (x-2)(x-1)$

a. Write a binomial that represents the height of the window.

$x-1$

b. The height of the door is 5 feet more than the height of the window. Write a polynomial that represents the height of the door.

$x-1+5$

$x+4$

<u>Practice: Solve. Keep x^2 positive!</u>