

# 10.1

## Graphing Square Root Functions

For use with Activity 10.1

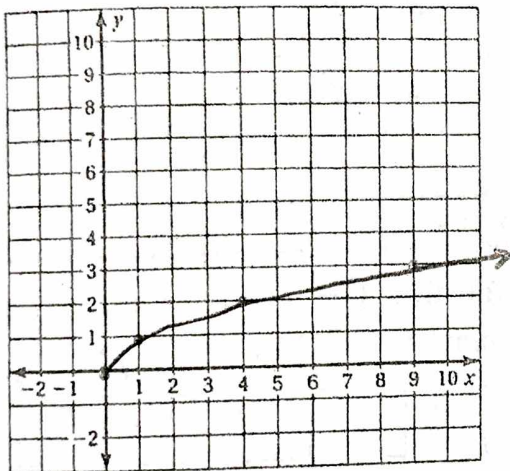
**Essential Question** How can you sketch the graph of a square root function?

### 1 ACTIVITY: Graphing Square Root Functions

Work with a partner.

- Make a table of values for the function.
- Use the table to sketch the graph of the function.
- Describe the domain of the function.
- Describe the range of the function.

a.  $y = \sqrt{x}$

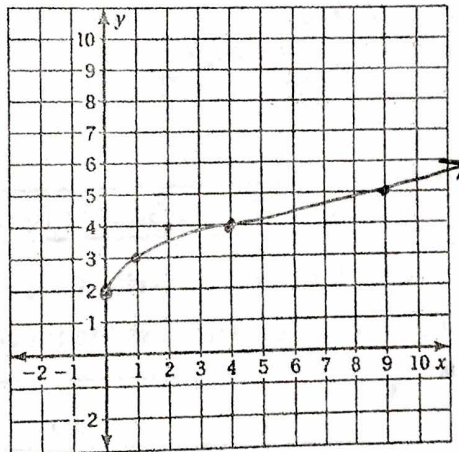


0	0
1	1
4	2
9	3

$$D: x \geq 0$$

$$R: y \geq 0$$

b.  $y = \sqrt{x} + 2$



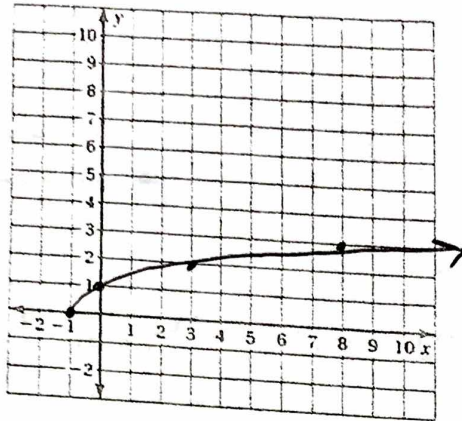
0	2
1	3
4	4
9	5

$$D: x \geq 0$$

$$R: y \geq 2$$

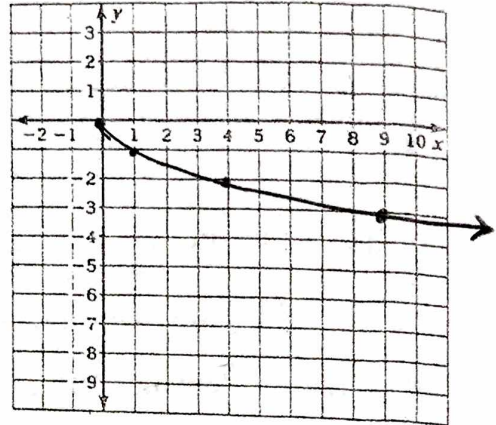
**10a** Graphing Square Root Functions (continued)

c.  $y = \sqrt{x+1}$



-1	0
0	1
3	2
8	3

d.  $y = -\sqrt{x}$



0	0
1	-1
4	-2
9	-3

**2** ACTIVITY: Writing Square Root Functions

Work with a partner. Write a square root function,  $y = f(x)$ , that has the given values. Then use the function to complete the table.

a.

$x$	$f(x)$
-4	0
-3	1
-2	1.4142
-1	1.7321
0	2
1	2.2361

$$f(x) = \sqrt{x+4}$$

b.

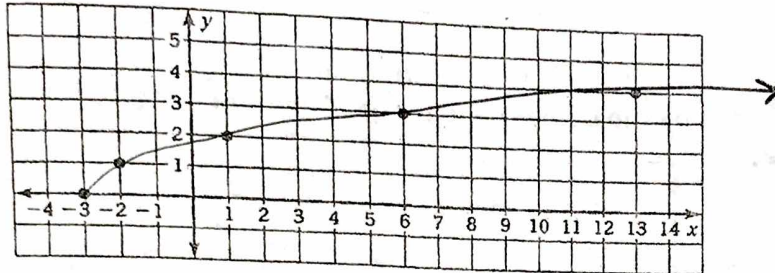
$x$	$f(x)$
-4	1
-3	2
-2	2.4142
-1	2.7321
0	3
1	3.2361

$$f(x) = \sqrt{x+4} + 1$$

## 10.1 Graphing Square Root Functions (continued)

### 3 ACTIVITY: Writing a Square Root Function

Work with a partner. Write a square root function,  $y = f(x)$ , that has the given points on its graph. Explain how you found your function.



$$y = \sqrt{x+3}$$

### What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you sketch the graph of a square root function? Summarize a procedure for sketching the graph. Then use your procedure to sketch the graph of each function.

a.  $y = 2\sqrt{x}$

grows twice as fast

b.  $y = \sqrt{x} - 1$

down 1

c.  $y = \sqrt{x-1}$

right 1

d.  $y = -2\sqrt{x}$

reflection over x-axis  
grows twice as fast

Name \_\_\_\_\_

**10.1****Practice**

For use after Lesson 10.1

Find the domain of the function.

1.  $y = 3\sqrt{x}$

$D: x \geq 0$

2.  $y = \sqrt{x-5}$

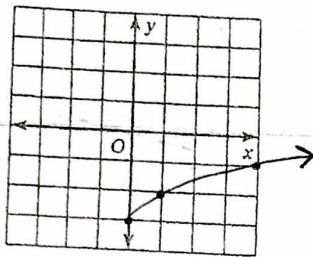
$D: x \geq 5$

3.  $y = 2\sqrt{-x+1}$

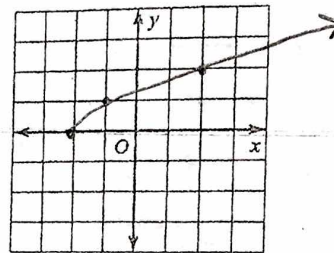
$D: x \leq -1$

Graph the function. Describe the domain and range. Compare the graph to the graph of  $y = \sqrt{x}$ .

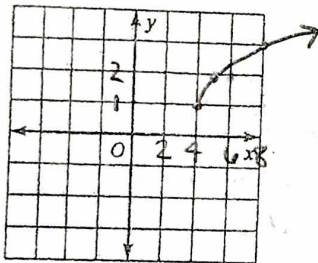
4.  $y = \sqrt{x} - 3$



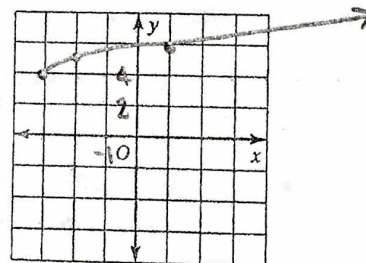
5.  $y = \sqrt{x+2}$



6.  $y = \sqrt{x-4} + 1$



7.  $y = -\sqrt{x+3} + 4$



8. The radius of a sphere is given by  $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$ , where  $S$  is the surface area of the sphere.

- a. Find the domain of the function. Use a graphing calculator to graph the function.

$D: x \geq 0$

- b. Use the *trace* feature to approximate the surface area of a sphere with a radius of 2 centimeters.

## Lesson #71b

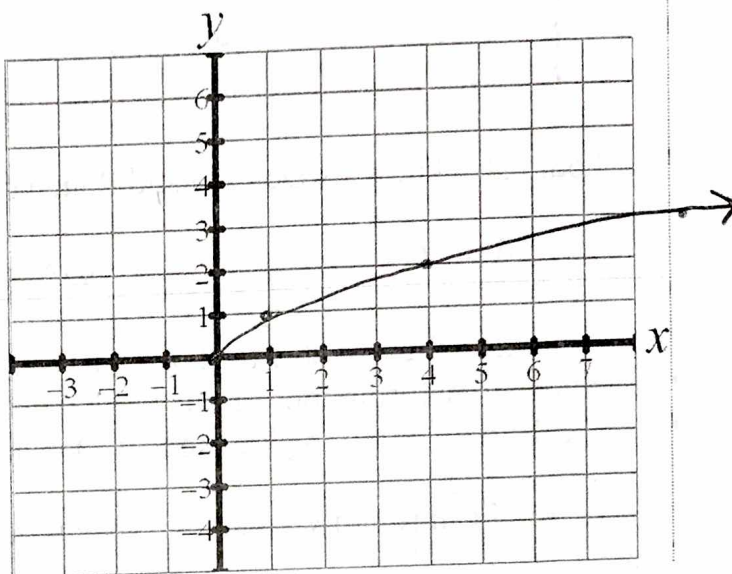
### Graphing Square Root Functions

Success Criteria: I can graph a square root function and determine the domain and range.

Success Criteria: I can determine what will happen to a graph if you add or subtract a number in different locations in a parabola.

**Problem #1:** (A) Use the home screen of your calculator to help complete the table to get some ordered pairs on the graph of the equation  $y = \sqrt{x}$ .

$x$	$y = \sqrt{x}$	$(x, y)$
-1	$\sqrt{-1}$	<del>(-1, )</del>
0	$\sqrt{0} = 0$	(0, 0)
1	$\sqrt{1} = 1$	(1, 1)
2	$\sqrt{2} = 1.4142$	(2, 1.4142)
2.5	$\sqrt{2.5} =$	(2.5, 1.6)
3	$\sqrt{3} = 1.7321$	(3, 1.7321)
4	$\sqrt{4} = 2$	(4, 2)



(B) Looking at the graph, what is the domain and range of the function  $y = \sqrt{x}$ ?

DOMAIN:  $x \geq 0$

RANGE:  $y \geq 0$

**Problem #2:** In previous chapters, you learned how the graph of a basic function (called the "parent" function) can be transformed/translated based on certain changes to the basic function.

In this section, the basic function (parent function) is  $y = \sqrt{x}$ .

(A) Consider the graph of  $y = \sqrt{x} - 2$ . How this will change the graph of  $y = \sqrt{x}$ ?

Prediction: down 2

Check your prediction by graphing  $y = \sqrt{x} - 2$  on your calculator.

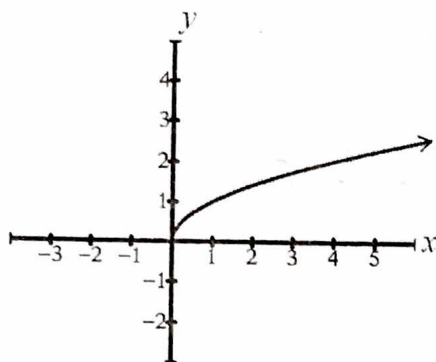
(B) Consider the graph of  $y = \sqrt{x-2}$ . How this will change the graph of  $y = \sqrt{x}$ ?

Prediction: right 2

Check your prediction by graphing  $y = \sqrt{x-2}$  on your calculator.

(1) Be familiar with what the graph of  $y = \sqrt{x}$  looks like:

It starts at the point  $(0, 0)$  and then rises gradually from left to right



**Note:** The graph is not a line!

It curves like a sideways half-parabola.

(2) You should also know its Domain and Range based on this graph:

DOMAIN:  $x \geq 0$

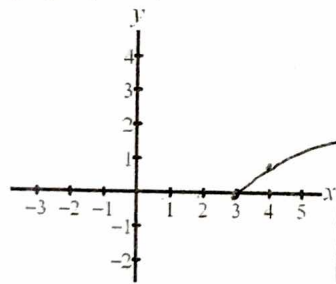
RANGE:  $y \geq 0$

(3) You should also then be able to ROUGHLY sketch the graph of equations which are translations/transformations of this basic function graph (using a calculator as needed), then use the new graph to give the new Domain and Range.

Success Criteria: I can roughly sketch a square root graph and give the domain and range based on an equation.

**Example 1:** Use the graph of the parent function  $y = \sqrt{x}$  to ROUGHLY graph the given function, then use your new graph to give the Domain and Range of the function.

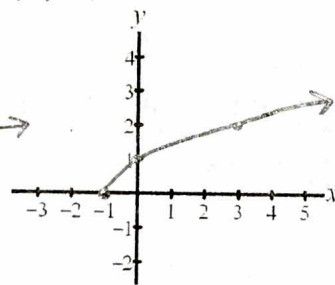
(A)  $y = \sqrt{x-3}$



DOMAIN:  $x \geq 3$

RANGE:  $y \geq 0$

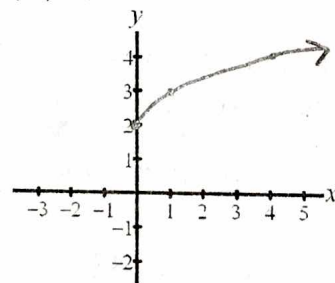
(B)  $y = \sqrt{x+1}$



DOMAIN:  $x \geq -1$

RANGE:  $y \geq 0$

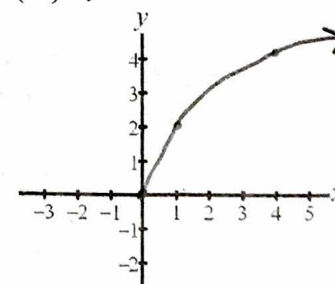
(C)  $y = \sqrt{x} + 2$



DOMAIN:  $x \geq 0$

RANGE:  $y \geq 2$

(D)  $y = 2\sqrt{x}$



DOMAIN:  $x \geq 0$

RANGE:  $y \geq 0$

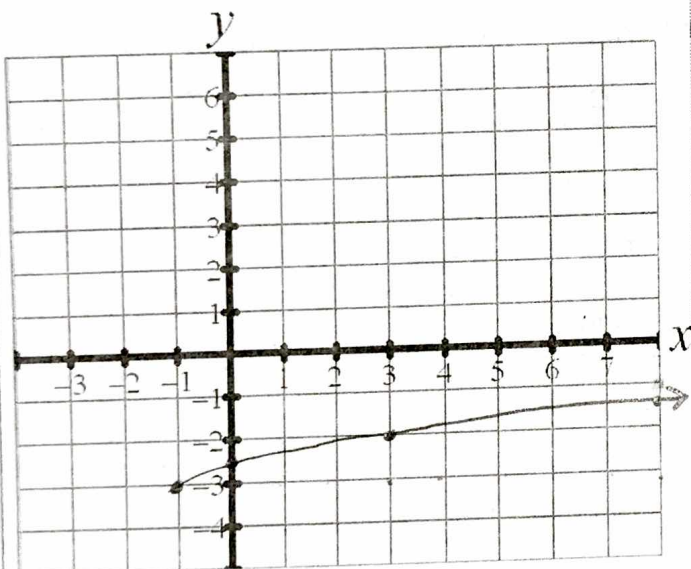
**Example 2:** Describe how the graph of the given function will be different from the graph of  $y = \sqrt{x}$ . Graph the given function and give the Domain and Range.

(A)  $y = 0.5\sqrt{x+1} - 3$

Describe any transformations/translations:

left 1, down 3, goes half as fast

Graph the function:



Give the domain and range of the function:

DOMAIN:  $x \geq -1$

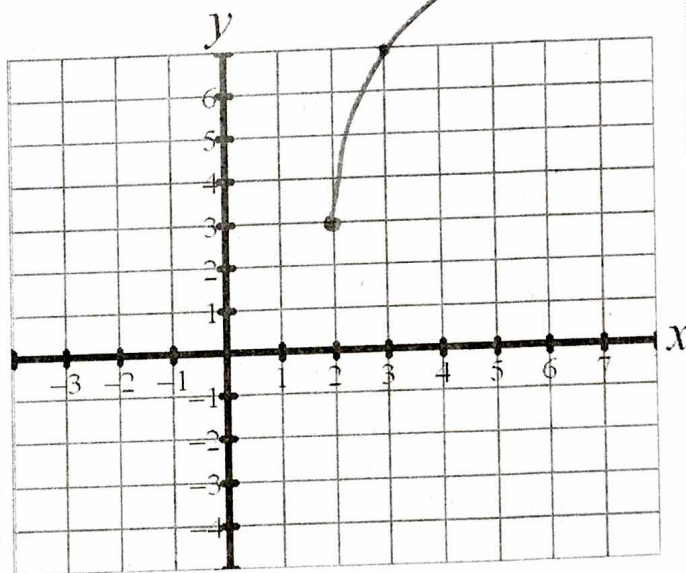
RANGE:  $y \geq -3$

(B)  $y = 4\sqrt{x-2} + 3$

Describe any transformations/translations:

right 2, up 3, goes 4 times as fast

Graph the function:



Give the domain and range of the function:

DOMAIN:  $x \geq 2$

RANGE:  $y \geq 3$

# 10.2

## Solving Square Root Equations

For use with Activity 10.2

**Essential Question** How can you solve an equation that contains square roots?

### ACTIVITY: Analyzing a Free-Falling Object

Work with a partner. The table shows the time  $t$  (in seconds) that it takes a free-falling object (with no air resistance) to fall  $d$  feet.

- a. Sketch the graph of  $t$  as a function of  $d$ .
- b. Use your graph to estimate the time it takes for a free-falling object to fall 240 feet.

3.9 seconds

- c. The relationship between  $d$  and  $t$  is given by the function

$$t = \sqrt{\frac{d}{16}}$$

Use this function to check the estimate you obtained from the graph.

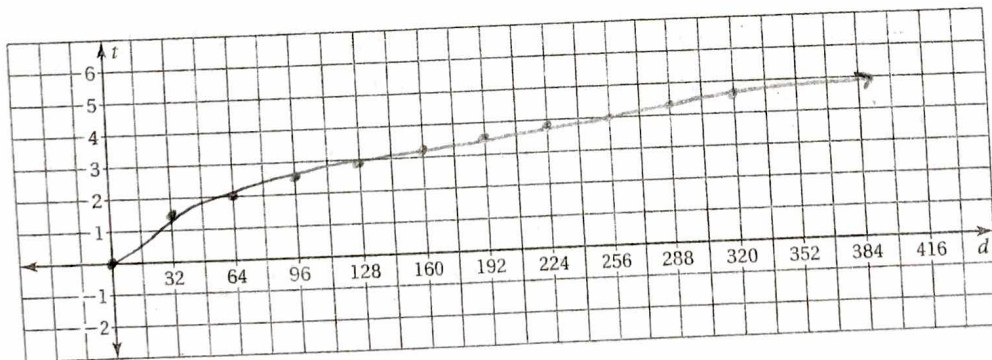
$$3.9^2 = \sqrt{\frac{d}{16}}^2$$

$$16 \cdot 15.21 = \frac{d}{16} \cdot 16 \quad d = 243.36$$

- d. Consider a free-falling object that takes 5 seconds to hit the ground. How far did it fall? Explain your reasoning.

384 ft?

$d$ feet	$t$ seconds
0	0.00
32	1.41
64	2.00
96	2.45
128	2.83
160	3.16
192	3.46
224	3.74
256	4.00
288	4.24
320	4.47





## 10.2

## Practice

For use after Lesson 10.2

Solve the equation. Check your solution.

$$1. \sqrt{x+4} = 9$$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$\sqrt{x} = 5$$

$$\boxed{x=25}$$

$$2. -2 = 9 - \sqrt{x}$$

$$\begin{array}{r} -6 \\ -6 \end{array}$$

$$-8 = -\sqrt{x}$$

$$8 = \sqrt{x}$$

$$\boxed{x=64}$$

$$3. 7 = \cancel{1} + 2\sqrt{x+4}$$

$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$6 = 2\sqrt{x+4}$$

$$\frac{6}{2} = \frac{2\sqrt{x+4}}{2}$$

$$3 = \sqrt{x+4}$$

$$9 = x+4$$

$$\boxed{x=5}$$

$$4. \sqrt{5x-11} - 3 = 5$$

$$\begin{array}{r} +3 \\ +3 \end{array}$$

$$\sqrt{5x-11} = 8$$

$$5x-11 = 64$$

$$5x = 75$$

$$\boxed{x=15}$$

$$5. \sqrt{4x-3} = \sqrt{x+6}$$

$$4x-3 = x+6$$

$$3x-3 = 6$$

$$3x = 9$$

$$\boxed{x=3}$$

$$6. \sqrt{8x+1} = \sqrt{7x+7}$$

$$8x+1 = 7x+7$$

$$x+1 = 7$$

$$\boxed{x=6}$$

$$7. x = \sqrt{12x-32}$$

$$\begin{array}{r} 32 \\ -8 \\ -12 \end{array} x^2 = 12x - 32$$

$$x^2 - 12x + 32 = 0$$

$$(x-8)(x-4) = 0$$

$$\boxed{x=8} \quad \boxed{x=4}$$

$$8. \sqrt{4x+13} = x-2$$

$$4x+13 = (x-2)^2$$

$$4x+13 = x^2 - 4x + 4$$

$$0 = x^2 - 8x - 9$$

$$0 = (x-9)(x+1)$$

$$\boxed{x=9}$$

$$\boxed{x=-1}$$

9. The formula  $\frac{S}{8} = \sqrt{df}$  relates the speed  $S$  (in feet per second), drag factor  $f$ ,

and distance  $d$  (in feet) it takes for a car to come to a stop after the driver applies

the brakes. A car travels at 80 feet per second and the drag factor is  $\frac{2}{3}$ . What

distance does it take for the car to stop once the driver applies the brakes?

$$\frac{80}{8} = \sqrt{\frac{2}{3}d}$$

$$10 = \sqrt{\frac{2}{3}d}$$

$$100 = \frac{2}{3}d$$

$$\boxed{d=150ft}$$

## Lesson #72

### Free-Falling Objects Intro

Success Criteria: I can use square root equations to model falling objects. I can use the free-falling objects equation to answer questions about situations.

#### Problem #1:

A stuffed monkey and a textbook are dropped from a high position. Predict who will hit the ground first.

The formula  $t = \sqrt{\frac{d}{16}}$  gives the distance  $d$  (in feet) an object free falls in  $t$  seconds.

$$y = \sqrt{\frac{x}{16}}$$

Problem #2: If our ceiling is 12 feet high, how long do the objects fall for?

$$t = \sqrt{\frac{12}{16}}$$

$$t = \sqrt{0.75}$$

$$t = 0.866$$

Problem #3: Galileo attempted to show that objects of different weights will still hit the ground at the same time when released from the same point. He wrote about dropping a ten-pound ball and a one-pound ball from the Leaning Tower of Pisa.

a) How tall is the Leaning Tower of Pisa in feet?

$$183 \text{ ft}$$

b) How long should it take an object to hit the ground dropped from the Tower?

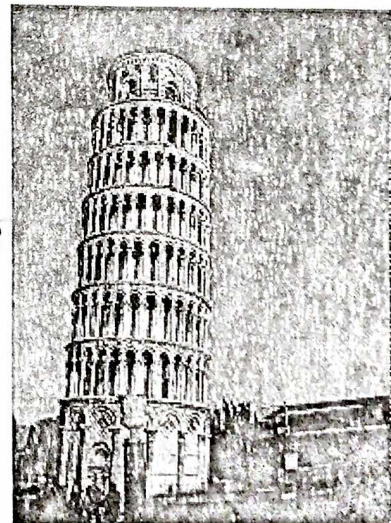
$$t = \sqrt{\frac{183}{16}}$$

$$t = \sqrt{11.44}$$

$$t = 3.38 \text{ sec}$$

c) Would every object hit the ground at the same time? Why or why not?

Ideally, yes but because of wind resistance it is not.



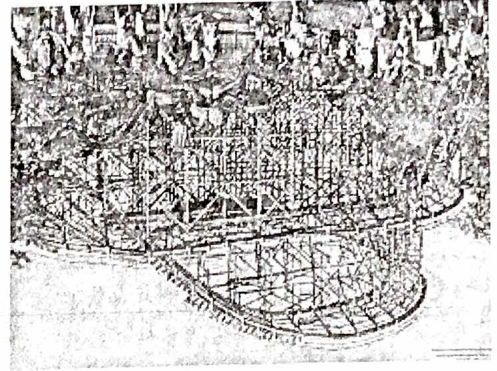
Video: "Braniac: Aristotle vs Galileo"

Do heavier objects fall faster than light objects?

Problem #4: When the Gemini opened at Cedar Point in 1978, it was the tallest, fastest, and steepest roller coaster in the world.

A penny dropped off its highest hill would hit the ground in 2.8 seconds. How high was the hill?

$$2.8 = \sqrt{\frac{d}{16}}$$
$$16 \cdot 7.84 = \frac{d}{16} \cdot 16$$
$$d = 125.44 \text{ ft}$$



Practice:

1. An object is dropped from a 56-foot bridge over a bay. How long will it take for the object to reach the water?

$$t = \sqrt{\frac{56}{16}}$$
$$t = \sqrt{3.5}$$
$$t = 1.87 \text{ sec}$$

2. An object dropped from the top of an observation tower takes 3.9 seconds to fall to the ground. How tall is the tower?

$$3.9 = \sqrt{\frac{d}{16}}$$
$$16 \cdot 15.21 = \frac{d}{16} \cdot 16$$
$$d = 243.36$$

3. A ball is dropped from a sixth-floor window and hits the ground after 2.1 seconds. What was the height of the window? Does that make sense?

$$2.1 = \sqrt{\frac{d}{16}}$$

$$16 \cdot 4.41 = \frac{d}{\cancel{16} \cdot \cancel{16}}$$

$$\boxed{d = 70.56}$$

4. An object falls from the top of a 100-foot communications tower. After how much time will the object hit the ground?

$$t = \sqrt{\frac{100}{16}}$$

$$t = \sqrt{6.25}$$

$$\boxed{t = 2.5 \text{ sec}}$$

## Square Root Applications

The formula  $t = \sqrt{\frac{d}{16}}$  gives the distance,  $d$  (in feet) an object free falls in  $t$  seconds.

1. What are the variables in this situation?

distance      time

2. What is the equation you will be using to answer questions?

$$t = \sqrt{\frac{d}{16}}$$

3. If an object falls 15 feet, how long will it take to fall?

- a. What variable is given to you? What does it equal?

$$d = 15$$

- b. What variable will you be solving for?

$$t = \text{time}$$

- c. Solve for the variable you identified in part b.

$$t = \sqrt{\frac{15}{16}}$$

$$t = \sqrt{0.9375}$$

$$t = 0.97 \text{ sec}$$

4. If an object falls for 1.6 seconds, how far did it fall?

- a. What variable is given to you? What does it equal?

$$\text{time} - t = 1.6$$

- b. What variable will you be solving for?

$$d$$

- c. Solve for the variable you identified in part b.

$$1.6 = \sqrt{\frac{d}{16}}$$

$$16 \cdot 2.56 = \frac{d}{16} \cdot 16$$

$$d = 40.96$$

## Square Root Applications

The time  $t$  (seconds) it takes you to swing forward and back on a rope is given by  $t = 6.28\sqrt{\frac{r}{32}}$ , where  $r$  is the rope length (in feet).

2. What are the variables in this situation?

$t = \text{time}$

$r = \text{length}$

3. What is the equation you will be using to answer questions?

$$t = 6.28\sqrt{\frac{r}{32}}$$

4. If you are swinging on a rope that is 9 feet long, how long will it take you to swing forward and back? (round to the nearest tenth)

- a. What variable is given to you? What does it equal?

$$r = 9$$

- b. What variable will you be solving for?

$$t = \text{time}$$

- c. Solve for the variable you identified in part b.

$$t = 6.28\sqrt{\frac{9}{32}}$$

$$t = 6.28\sqrt{0.1875}$$

$$t = 6.28 \cdot 0.433$$

$$t = 2.72 \text{ sec}$$

5. If it takes you 16 seconds to swing forward and back, how long is the rope?

- a. What variable is given to you? What does it equal?

$$t = 16$$

- b. What variable will you be solving for?

✓

- c. Solve for the variable you identified in part b.

$$\frac{16}{6.28} = \frac{6.28\sqrt{\frac{r}{32}}}{6.28}$$

$$2.548 = \sqrt{\frac{r}{32}}$$

$$32 \cdot 6.49 = \frac{r}{32} \cdot 32$$

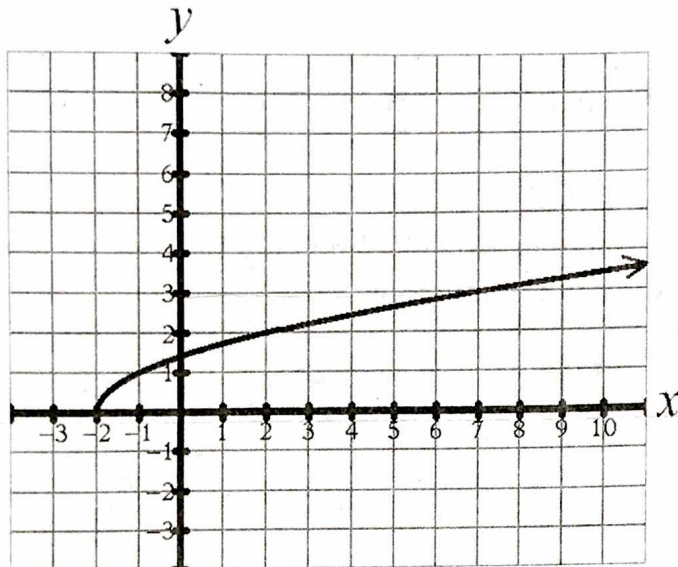
$$r = 207.72 \text{ ft}$$

**Lesson #72b**  
**Solving Square Root Equations**

Success Criteria: I can solve a square root equations using the graphs and undoing operations.

**Problem #1:**

(A) The graph of the function  $f(x) = \sqrt{x+2}$  is given below.



Use the graph to find the value of  $x$  for which  $f(x) = 3$ .

Answer:  $x = 7$

(B) Another way to find the value of  $x$  for which  $f(x) = 3$  for the function  $f(x) = \sqrt{x+2}$  is to simply replace the  $f(x)$  with 3 in the equation to get  $3 = \sqrt{x+2}$ , and then solve for  $x$ .

Algebraically UNDO operations connected to the  $x$  in the equation  $3 = \sqrt{x+2}$  to solve for  $x$ . (Show all work!)

$$\begin{aligned} 3^2 &= \sqrt{x+2}^2 \\ 9 &= x+2 \\ -2 & \quad -2 \\ x &= 7 \end{aligned}$$

(C) Did you get the same answer in parts (A) and (B)? Yes

## Solving Square Root Equations:

We can solve an equation that contains a square root by UNDOing operations connected to the variable. There are 2 key steps, however, when solving a square root equation.

- (1) Get the  $\sqrt{\quad}$  alone on one side before squaring it to UNDO this operation.
- (2) You MUST always check your solutions, as often squaring both sides of an equation leads to apparent solutions which actually do not work (These are called Extraneous Solutions)

**Example 1:** Solve each square root equation algebraically by UNDOing operations. Check your solutions to make sure they are not extraneous.

(A)  $\sqrt{x-7} = 3$   
 $\quad +7 \quad +7$   
 $\sqrt{x-7+7} = 3+3$   
 $\sqrt{x} = 6$   
 $\sqrt{x}^2 = 6^2$   
 $x = 36$

(B)  $4 = 8 + \sqrt{x-5}$   
 $\quad -8 \quad -8$   
 $-4 = \sqrt{x-5}$   
 $(-4)^2 = \sqrt{x-5}^2$   
 $16 = x-5$   
 $\quad +5 \quad +5$   
 $x = 21$

(C)  $4\sqrt{x+1} + 3 = 19$   
 $\quad -3 \quad -3$   
 $4\sqrt{x+1} = 16$   
 $\quad \div 4 \quad \div 4$   
 $\sqrt{x+1} = 4$   
 $\sqrt{x+1}^2 = 4^2$   
 $x+1 = 16$   
 $\quad -1 \quad -1$   
 $x = 15$

(D)  $\sqrt{2x-1} - \sqrt{x+4} = 0$   
 $\quad +\sqrt{x+4} \quad +\sqrt{x+4}$   
 $\sqrt{2x-1} = \sqrt{x+4}$   
 $2x-1 = x+4$   
 $\quad -x \quad -x$   
 $x-1 = 4$   
 $\quad +1 \quad +1$   
 $x = 5$

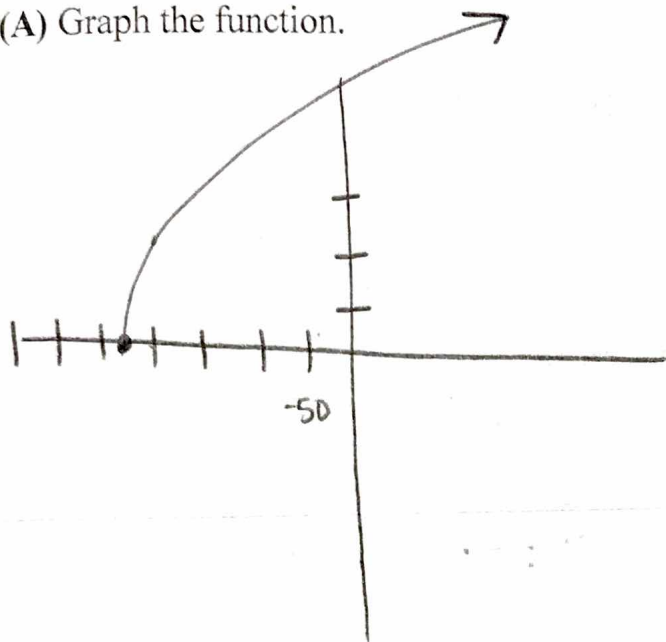


Success Criteria: I can answer questions about real life situations using square root equations.

**Example 2:** The speed of sound  $s$  (in meters per second) through air is given by

$$s = 20\sqrt{T + 273}, \text{ where } T \text{ is the temperature in degrees Celsius.}$$

(A) Graph the function.



(B) Algebraically find the temperature when the speed of sound is 340 meters per second. Check using a calculator.

$$\frac{340}{20} = \frac{20\sqrt{T+273}}{20}$$

$$17 = \sqrt{T+273}$$

$$\begin{array}{r} 289 = T + 273 \\ -273 \quad -273 \\ \hline \end{array}$$

$$\boxed{116 = T}$$

(C) A starting gun fires to begin a race at many track meets. Will spectators hear the sound for the beginning of the race more accurately in cold weather or warm weather?

warmer weather

You are in an Olympic stadium a distance of 160 meters from the starting gun.

(D) What is the speed of sound when the temperature is  $-17^\circ$  Celsius?

$$s = 20\sqrt{-17+273}$$

$$s = 20\sqrt{256}$$

$$s = 20 \cdot 16$$

$$\boxed{s = 320 \text{ m/s}}$$

(E) Using the equation  $D = rt$  (Distance equals rate times time), how long will the runners be running before you hear the sound of the gun?

$$\frac{160}{320} = \frac{320t}{320}$$

$$\boxed{t = 0.5 \text{ m}}$$

# Lesson #73

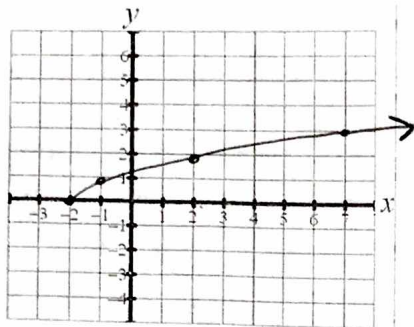
## Square Root and Cumulative Review

Examples: Graph the function. Describe the domain and range.

$$y = \sqrt{x+2}$$

Domain:  $x \geq -2$

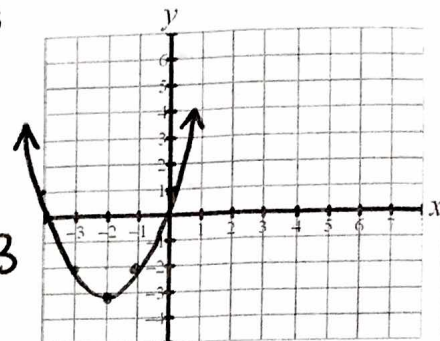
Range:  $y \geq 0$



$$y = (x+2)^2 - 3$$

Domain:  $\mathbb{R}$

Range:  $y \geq -3$

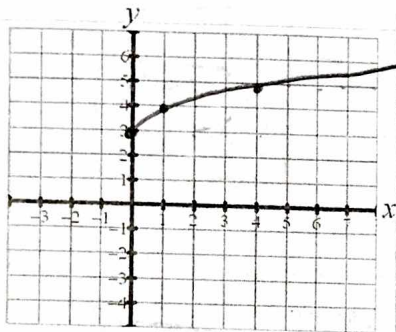


Practice: Graph the function. Describe the domain and range.

1.  $y = \sqrt{x} + 3$

Domain:  $x \geq 0$

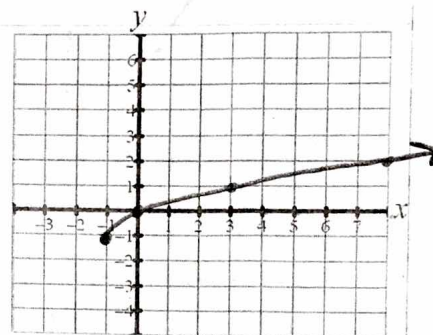
Range:  $y \geq 3$



2.  $y = (x+1)^2 - 1$

Domain:  $x \geq -1$

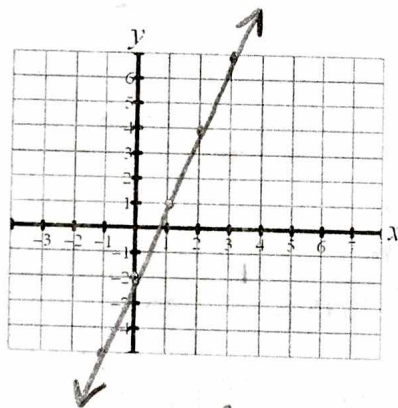
Range:  $y \geq -1$



3.  $y = 3x - 2$

Domain:  $\mathbb{R}$

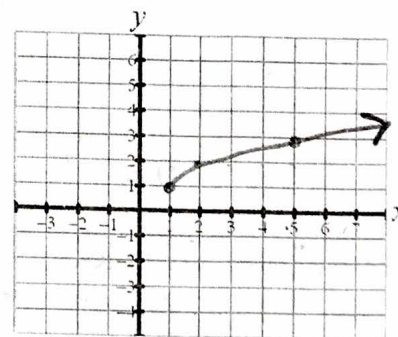
Range:  $\mathbb{R}$



4.  $y = \sqrt{x-1} + 1$

Domain:  $x \geq 1$

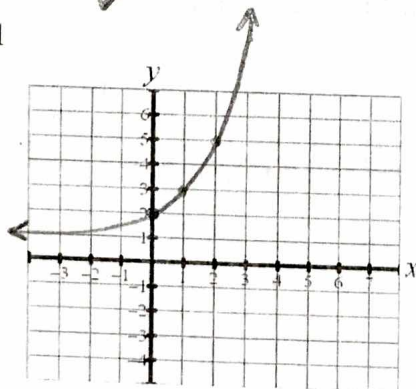
Range:  $y \geq 1$



5.  $y = 2^x + 1$

Domain:  $\mathbb{R}$

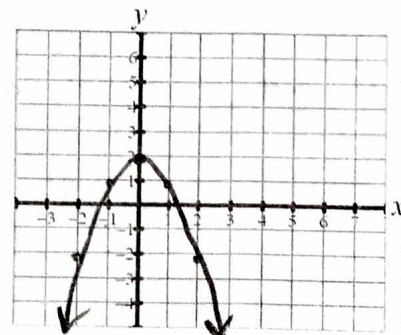
Range:  $y > 1$



6.  $y = -x^2 + 2$

Domain:  $\mathbb{R}$

Range:  $y \leq 2$



Examples: Solve the equation. Check your solution(s) for problems with radicals.

$$\sqrt{x-7}-4=1$$

$$\begin{array}{r} +4 \\ +4 \end{array}$$

$$\sqrt{x-7}=5$$

$$x-7=25$$

$$\begin{array}{r} +7 \\ +7 \end{array}$$

$$\boxed{x=32}$$

$$x^2-2x=8$$

$$\begin{array}{r} -8 \\ -8 \end{array}$$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$$\boxed{x=4} \quad \boxed{x=-2}$$

Solve the equation. Check your solution(s) for problems with radicals.

7.  $\sqrt{x-2}=12$

$$\begin{array}{r} +2 \\ +2 \end{array}$$

$$\sqrt{x}=14$$

$$\boxed{x=196}$$

8.  $\sqrt{x-2}+4=5$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$\sqrt{x-2}=1$$

$$x-2=1$$

$$\boxed{x=3}$$

9.  $\sqrt{x-2}=\sqrt{3x}$

$$x-2=3x$$

$$-2=2x$$

$$\boxed{x=-1}$$

10.  $4x-3=-23$

$$\begin{array}{r} +3 \\ +3 \end{array}$$

$$4x=-20$$

$$\boxed{x=-5}$$

11.  $x^2-x=6$

$$\begin{array}{r} -6 \\ -6 \end{array}$$

$$x^2-x-6=0$$

$$(x-3)(x+2)=0$$

$$\boxed{x=3} \quad \boxed{x=-2}$$

12.  $\frac{14}{\sqrt{x}}=7$

$$\begin{array}{r} -14 \\ -14 \end{array}$$

$$-\sqrt{x}=-7$$

$$\sqrt{x}=7$$

$$\boxed{x=49}$$

13.  $\sqrt{x+3}=\sqrt{2x-8}$

$$x+3=2x-8$$

$$3=x-8$$

$$\boxed{x=11}$$

14.  $\frac{10}{\sqrt{x}}-3=4$

$$\begin{array}{r} -10 \\ -10 \end{array}$$

$$-3\sqrt{x}=-6$$

$$\sqrt{x}=2$$

$$\boxed{x=4}$$

**Lesson #74**  
**Inverse of a Function**

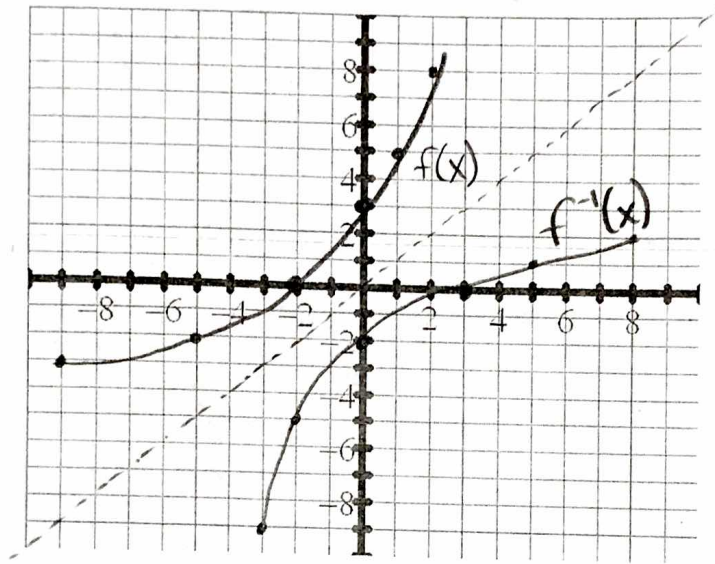
Success Criteria: I can graph an inverse function given the coordinates or a graph of a function. I can determine if a line is reflected over the line  $y=x$ .

**Problem #1:**

(A) Several ordered pairs  $(x, f(x))$  for a function  $f(x)$  are given below.

Ordered pairs for  $f(x)$ :  $(-9, -3)$ ,  $(-5, -2)$ ,  $(-2, 0)$ ,  $(0, 3)$ ,  $(1, 5)$ ,  $(2, 8)$

Plot the ordered pairs and then connect them with a smooth curve to graph the function  $f(x)$ .



(B) To get the INVERSE of the function, denoted by  $f^{-1}(x)$ , we switch the  $x$  and  $y$  coordinates for all ordered pairs.

For each ordered pair for  $f(x)$  given in part (A), switch the  $x$  and  $y$  coordinates to find several ordered pairs on the graph of  $f^{-1}(x)$ .

Ordered pairs for  $f^{-1}(x)$ :  $(-3, -9)$ ,  $(-2, -5)$ ,  $(0, -2)$ ,  $(3, 0)$ ,  $(5, 1)$ ,  $(8, 2)$

(C) On the coordinate plane in part (A), plot these ordered pairs and then connect them with a smooth curve to graph the function  $f^{-1}(x)$ .

(D) Label this new graph as  $f^{-1}(x)$  and label the original graph as  $f(x)$ .

(E) What do you notice about the graphs of  $f(x)$  and  $f^{-1}(x)$ ?

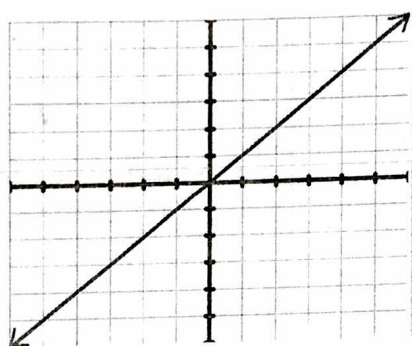
Reflections

(F) On the coordinate plane in part (A), draw the graph of the linear equation  $y = x$ . Describe how you can use the graph of  $f(x)$  and the line  $y = x$  to draw the graph of  $f^{-1}(x)$ .

The **INVERSE** of a function:

(A) You can get the graph of the **INVERSE function**  $f^{-1}(x)$  for a function  $f(x)$ , by **switching** the Domain (the  $x$ -values) and the Range (the  $y$ -values).

(B) This will result in the graph of  $f^{-1}(x)$  being the reflection of the graph of  $f(x)$  over the line  $y = x$ .



Example 1: For each given representation of  $f(x)$ , find the inverse.

(A)

$x$	-2	-1	0	1
$y$	3	5	8	10

Inverse:

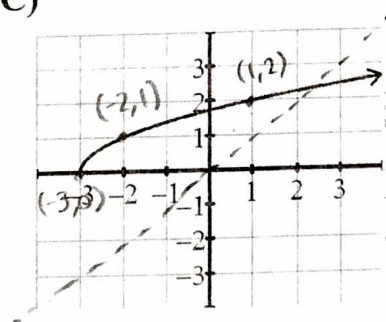
$x$	3	5	8	10
$y$	-2	-1	0	1

(B)  $(-1, 0)$ ,  $(0, 4)$ ,  $(1, 5)$ ,  
 $(3, 6)$ ,  $(8, 8)$

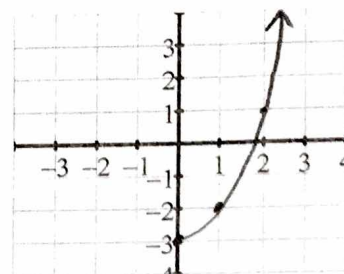
Inverse:

$(0, -1)$ ,  $(4, 0)$ ,  
 $(5, 1)$ ,  $(6, 3)$ ,  
 $(8, 8)$

(C)



Inverse:



Example 2:

(A) Fill in the table for  $f(x) = 2x - 6$  using a calculator.

$x$	3	4	5	6
$y$	0	2	4	6

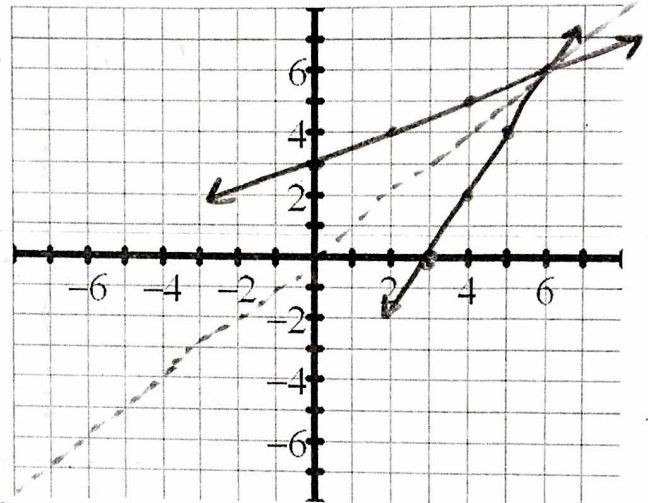
Fill in the table for  $f^{-1}(x) = \frac{1}{2}x + 3$

$x$	0	2	4	6
$y$	3	4	5	6

What do you notice?

They are inverses

(B) Graph both  $f(x)$  and  $f^{-1}(x)$  to check that they are reflections of each other over the line  $y = x$ .



## Lesson #75 Inverse of a Function

Warm-up: For each given representation of  $f(x)$ , find the inverse.

(A)

$x$	-2	-1	0	1
$y$	5	7	9	11

Inverse:

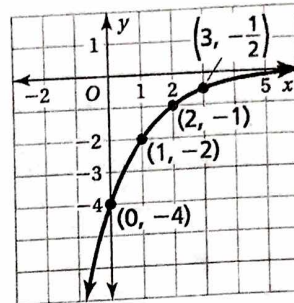
$x$	5	7	9	11
$y$	-2	-1	0	1

(B)  $(-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8)$

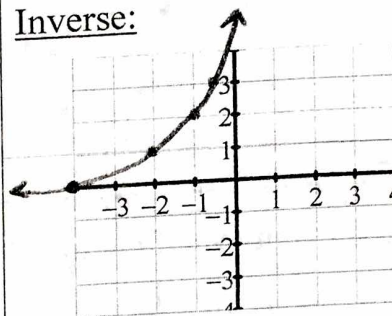
Inverse:

$(8, -2), (2, -1), (0, 0), (2, 1), (8, 2)$

(C)



Inverse:



Success Criteria: I can find the equation for the inverse of a function. I can graph the function and its inverse on the same graph.

How to find an inverse function

Step One	change $f(x)$ to $y$
Step Two	flip $x$ and $y$
Step Three	solve for $y$

Example 1:

(A) Find the inverse function  $f^{-1}(x)$  for

$$f(x) = -2x + 5$$

$$y = -2x + 5$$

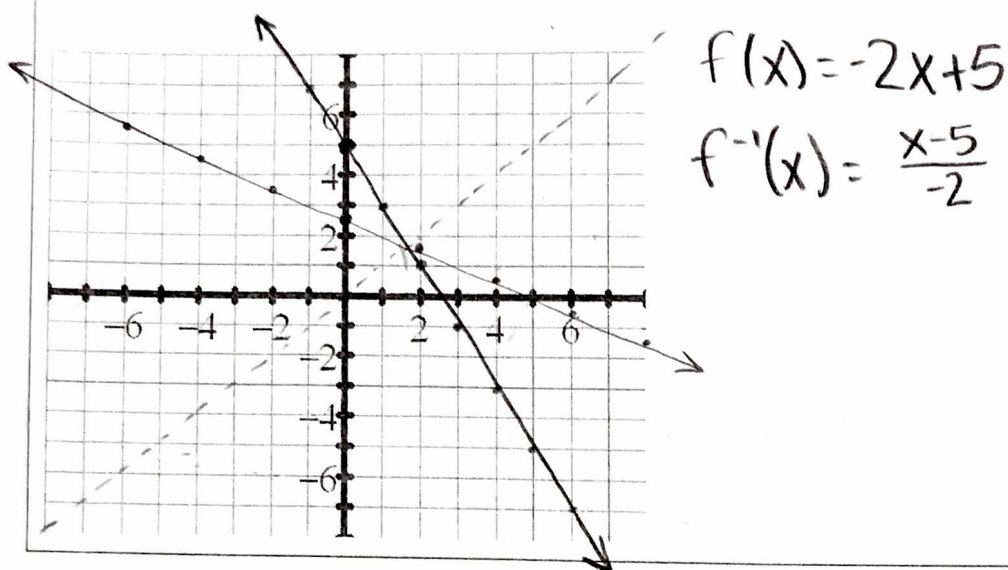
$$x = \frac{-y + 5}{-2}$$

$$\frac{x - 5}{-2} = \frac{-y}{-2}$$

$$y = \frac{x - 5}{-2}$$

$$f^{-1}(x) = \frac{x - 5}{-2}$$

(B) Graph both  $f(x)$  and  $f^{-1}(x)$  to check that they are reflections of each other over the line  $y=x$ .



Example 2:

(A) Find the inverse function  $f^{-1}(x)$  for

$$f(x) = \frac{4}{x}$$

$$y = \frac{4}{x}$$

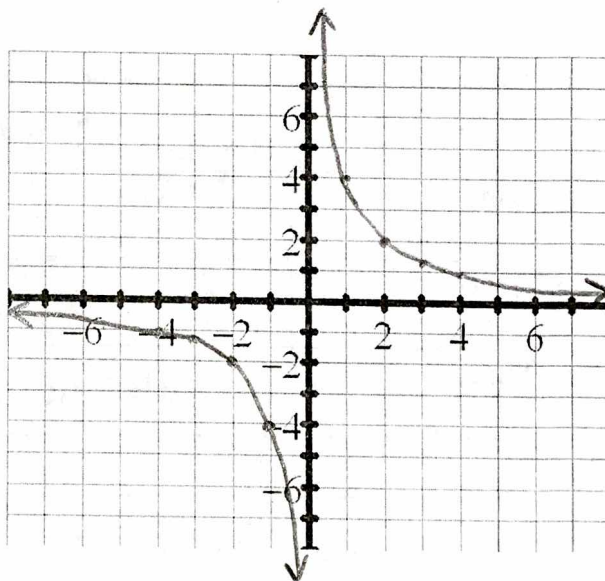
$$y \cdot x = \frac{4}{y} \cdot y$$

$$\frac{yx}{x} = \frac{4}{x}$$

$$y = \frac{4}{x}$$

$$f^{-1}(x) = \frac{4}{x}$$

(B) Graph both  $f(x)$  and  $f^{-1}(x)$  to check that they are reflections of each other over the line  $y=x$ .





Example 3:

(A) Find the inverse function  $f^{-1}(x)$  for

$$f(x) = x^2 + 3 \text{ where } x \geq 0$$

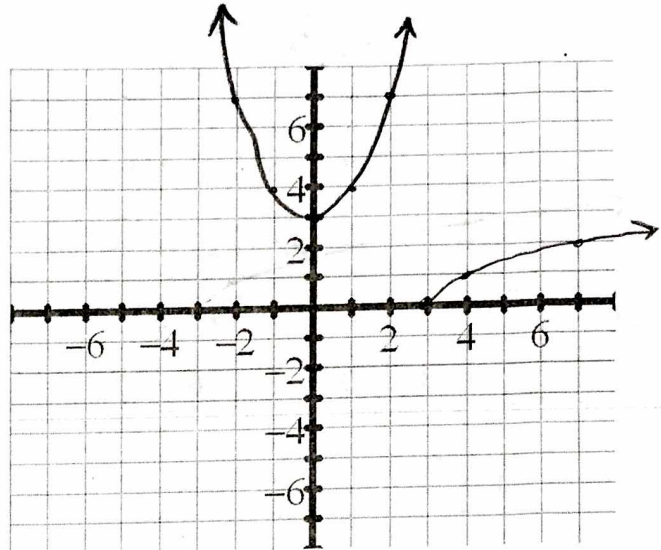
$$y = x^2 + 3$$

$$x = \frac{y^2 + 3}{-3}$$

$$x - 3 = y^2$$

$$y = \sqrt{x - 3}$$

(B) Graph both  $f(x)$  and  $f^{-1}(x)$  to check that they are reflections of each other over the line  $y = x$ .



**Lesson #75b**  
**Celsius/Fahrenheit Conversion**

1. What is water's freezing point in degrees Fahrenheit and degrees Celsius?

$32^{\circ}\text{F}$                        $0^{\circ}\text{C}$

2. What is water's boiling point in degrees Fahrenheit and degrees Celsius?

$212^{\circ}\text{F}$                        $100^{\circ}\text{C}$

3. What is the normal human body temperature in degrees Fahrenheit and degrees Celsius?

$98.6^{\circ}\text{F}$                        $37^{\circ}\text{C}$

The conversion formulas for Celsius/Fahrenheit are inverse functions

When you know the temperature in Celsius, you convert to Fahrenheit using:  $F = \frac{9}{5}C + 32$

4. Using the equation for Fahrenheit, rewrite the equation for C.

$$F = \frac{9}{5}C + 32$$

$$\begin{array}{r} -32 \\ -32 \end{array} \quad \begin{array}{r} -32 \\ -32 \end{array}$$

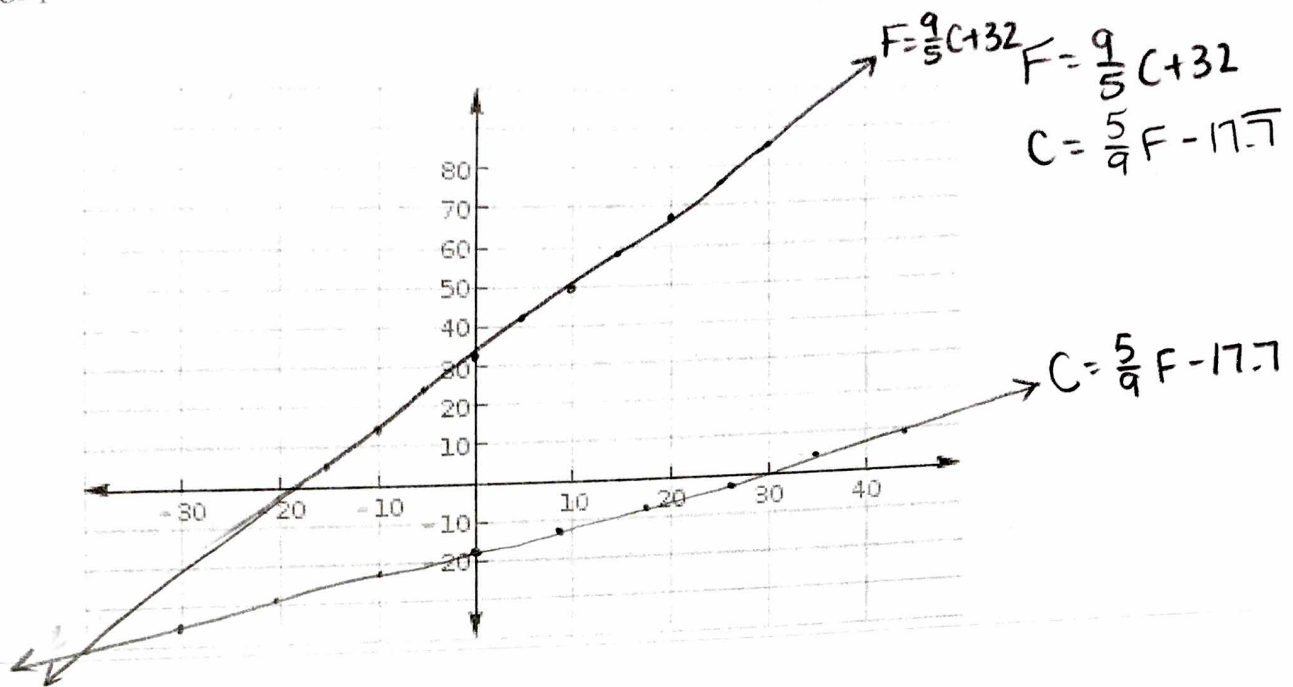
$$F - 32 = \frac{9}{5}C$$

$$C = \frac{5}{9}(F - 32)$$

5. Complete the chart below using those formulas:

Celsius	Fahrenheit
0	$32^{\circ}$
$4.\bar{4}$	40
16	$60.8$
$23.\bar{8}$	75
30	$86$
$37$	98.6
-40	$-40$
$-17.\bar{7}$	0

c. Graph the formulas for F and C at the same time below. What do you notice?



7. At what temperature (in degrees Celsius) would it have to be outside before you wore a winter coat?
8. At what temperature (in degrees Celsius) would you want it to be outside before you went swimming?
9. If it is 20 degrees Celsius outside, how would you describe the temperature - Hot, warm, cool, or cold?