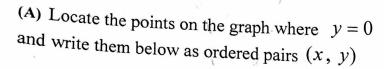
#### Lesson #67

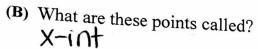
#### Solving Quadratic Functions by Graphing

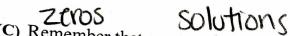
Success Criteria: I can find the zeros of an equation by graphing, the table and factoring.

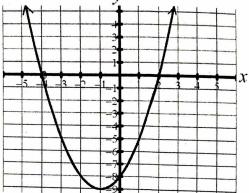
**Problem #1:** The graph of the equation  $y = x^2 + 2x - 8$  is given below.



$$(-4,0)$$
  $(2,0)$ 







(C) Remember that every ordered pair on the graph makes the equation  $y = x^2 + 2x - 8$  true. Choose one of the points from part (A) and show that it makes the equation true:

$$y = x^{2} + 2x - 8$$

$$0 = (-4)^{2} + 2(-4) - 8$$

$$0 = 16 - 8 - 8$$

$$0 = 0$$

**(D)** Graph the function on a calculator and look at the table. When does y = 0?

$$-4$$
 and  $2$ 

(E) Find the zeros of  $y = x^2 + 2x - 8$  by factoring. What do you see? 0 = 1 b = 2 c = 8 y = (x+4)(x-2)



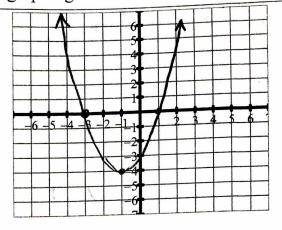
So far you have learned 3 methods for solving quadratic functions. After setting the equation equal to zero, you can:

- 1. Solve by <u>factoring</u>
- 2. Solve by araphing
- 3. Solve by table

Example #1: Solve the equation  $x^2 + 2x = 3$  by graphing.

$$x^2 + 2x = 3$$
  
 $x^2 + 2x - 3 = 0$ 

Solutions: 
$$x = -3$$
 and  $x = 1$ 



Success Criteria: I can find the zeros on a calculator using the "zero" feature.

To find the x-intercepts (or zeros) of a graph using a graphing calculator:

While on the graph screen,

STEP #1: Press (2ND) then press (TRACE) then choose "Zero"

STEP #2: Move the cursor to set a Left Bound then press ENTER ,

then set a Right Bound then press ENTER

and then press ENTER again.

Example #2: Use a graphing calculator to find the solutions of the equation.

(A) 
$$x^2 + 24 = -10$$
  
 $x^2 + 24x + 10 = 0$ 

(B) 
$$x^2 - 10x = -16$$
  
 $\chi^2 - 10\chi + 10 = 0$ 

Solutions:

Solutions: (8,0) (2,0)

Example #3: Use two methods to find the solutions of  $x^2 + 6 = -7x$ .

Method 1: Factoring

$$x^{2}+6=-7x$$
  
 $x^{2}+7x+6=0$   
 $(x+6)(x+1)=0$ 

Solutions: 
$$x = -Q$$
 and  $x = -1$ 

$$x = -1$$

Method 2: Graphing calculator

$$x^2 + 6 = -7x$$

Graph: 
$$y_1 = \chi^2 + \chi + \zeta$$

Solutions: 
$$x = -(a \text{ and } x = -(a \text{$$

Example #4: Use your graphing calculator to determine HOW MANY solutions the equation will have (0, 1, or 2). You don't need to find the solutions.

$$-x^2 = 4x - 5$$
$$0 = X^2 + 4x - 5$$

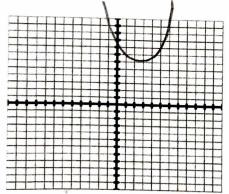
$$x^2 = -4$$

$$x^2 + 4 = 0$$

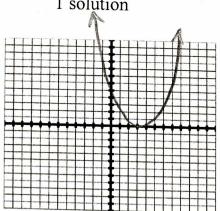
solutions

Example #5: Sketch a graph with the given number of solutions:

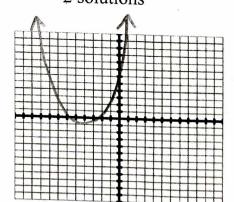




1 solution



2 solutions



Success Criteria: I can write an equation to model projectile motion. I can use the calculator to answer questions

Example #6: A football player kicks a football 2 feet above the ground with an initial upward velocity of 75 feet per second.

(A) The equation  $h = -16t^2 + vt + s$  will give the height h of the ball (in feet) after t seconds. The variable v represents the initial velocity (positive if up and negative if down) and the variable s represents the initial height of the ball.

Write the equation:  $h = -10t^2 + 75t + 2$ 

(B) After how many seconds is the football 50 feet above the ground?

0.76 Seconds 3.92 seconds

(C) After how many seconds does the football hit the ground?

4.71 seconds

# Lesson #68

Solving Quadratic Equations Using Square Roots

Success Criteria: I can solve a quadratic equation using square roots.

(A) EVALUATE 
$$3(5)^2 + 4$$
  
 $3(25) + 4$ 

$$3x^2 + 4 = 52$$
  $x^2 = 16$   
 $3x^2 = 48$   $x = 4, -4$ 

$$X=16$$

$$X=4-4$$

You need to DO the operations in problem (A) to find the value.

You need to UNDO the operations in problem (B) to solve for the variable x.

Follow the order of operations (PEMDAS) when we DO operations.

Follow the order of operations in reverse order (SADMEP) when we UNDO operations.

State the order in which you would undo the operations that are connected to the variable xin the equation  $3x^2 + 4 = 52$ :

First undo: +4

Then undo: 3

Then undo: 2 (exponent)

Now go through the process of undoing the operations to solve for the variable x:

$$3x^{2} + 4 = 52$$
 $4$ 
 $3x^{2} - 48$ 

Example #2: Solve the equation by UNDOing operations.

$$4x^{2} - 3 = 31$$

$$+5 +5$$

$$4x^{2} - 36$$

$$4$$

$$X^{2} - 9$$

$$X = 9$$

$$X = 9$$

$$X = 36$$

$$7x^{2}-10=-10$$

$$+10$$

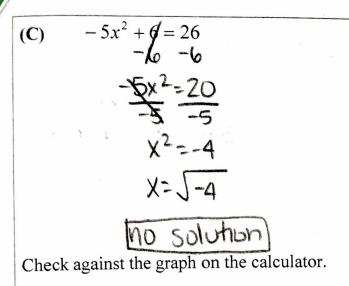
$$7x^{2}=0$$

$$7$$

$$X^{2}=0$$

$$X=0$$

$$X=0$$



(D) 
$$\frac{4(x-3)^2 = 100}{4}$$
  
 $\sqrt{(x-3)^2 = 125}$   
 $X-3=\pm 5$   
 $\sqrt{3}=5$   
 $\sqrt{3}=5$ 

<u>Note:</u> Because the quadratic equation  $5x^2 + 2x - 6 = 0$  contains more than one variable term that cannot be combined, we do not have one variable term to UNDO operations on.

Therefore, we cannot UNDO operations to solve such quadratic equations.

Success Criteria: I can tell by looking at an equation if it can be solved using a square root.

Example #2: For each equation, decide if it can be solved by UNDOing operations. Write "YES" or "NO" for each equation.

(A) 
$$3x^2 - 2x + 7 = 0$$
 (B)  $x^2 - 2 = 5$  (C)  $x^2 - 2x = 5$  (D)  $4x - 1 = 5x^2$  (E)  $4x^2 - 2 = 15$  (F)  $3x^2 = 6$ 

## Lesson #68b Review: Solving Quadratic Functions

1. Solve: 
$$(x-3)(x+7)=0$$

Solve: 
$$(x-3)(x+7)=0$$
  
 $X-3=0$   
 $+3+3$   
 $X=3$   
 $X=7$ 

3. Solve by factoring: 
$$6m^2 + 9m = 0$$

$$3m(2m+3)=0$$
 $3m=0$   $2m+3=0$ 
 $3m=0$   $2m+3=0$ 
 $3m=-3$ 
 $2m=-3$ 
 $2m=-3$ 
 $2m=-3$ 
 $2m=-3$ 

5. Solve by factoring: 
$$y^2 - 25 = 0$$

$$(y-5)(y+5)=0$$
  
 $y-5\neq0$   $y+5=0$   
 $+5+5$   $-5=5$   
 $y=5$   $y=5$ 

7. Solve by factoring: 
$$3x^2 - 7x = -2$$

Foring: 
$$3x^2 - 7x = -2$$
  
 $3x^2 - 7x + 2 = 0$   
 $(x - 6)(x - 1) = 0$   
 $(x - 2)(3x - 1) = 0$ 

2. Solve using a calculator: 
$$v^2 + 3v = 4$$

$$X=1$$
  $X=-4$ 

4. Solve by factoring: 
$$x^2 - 13x = -42$$

$$X^{2}-13x+42=0$$
  
 $(X-7)(X-6)=0$   
 $X-7=0$   $X-6=0$   
 $X=7=0$   $X=6=0$   
 $X=7=0$   $X=6=0$ 

**6.** Solve by taking square roots: 
$$y^2 - 25 = 0$$

$$y^2 = 25$$
  
 $y = \sqrt{25}$   
 $y = \pm 5$ 

**8.** Solve by any method: 
$$4x^2 + 13x + 3 = 0$$

thod: 
$$4x^2 + 13x + 3 = 0$$
  
 $(x+12)(x+1) = 0$   
 $(x+3)(4x+1) = 0$ 

# Solve by the given method. Would there be a better method for any of the problems?

1. Factoring: 
$$x^2 + 2x = 35$$

$$\begin{array}{c} X + 2x = 35 \\ X^{2} + 2x - 35 = 0 \\ (X + 7)(X - 5) = 0 \\ X + 7 = 0 \quad X - 5 = 0 \\ 17 = 7 \quad 45 + 5 \\ \hline X = -7 \quad X = 5 \end{array}$$

2. Calculator: 
$$x^2 - 7x + 10 = 0$$

Factoring 
$$(x-5)(x-2)=0$$
  
 $x-5=0$   $x-2=0$   
 $15.15$   $12.12$ 

3. **Factoring:** 
$$x^2 - 4 = 0$$

3. Factoring: 
$$x^2-4=0$$
  
Square  $(x-2)(x+2)=0$ 

$$X-2=0$$
  $X+2=0$   $X=-2$ 

$$X^{2} = 4$$

$$X = \sqrt{4}$$

$$X = \pm 2$$

4. Square Roots: 
$$25x^2 - 36 = 0$$

5. **Factoring:** 
$$3x^2 + 2x - 8$$

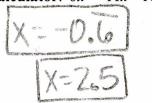
$$(x+6)(x-4)$$

$$(x+2)(3x-4)$$

$$\frac{3x=4}{3}$$

$$\begin{bmatrix} 3X=4 \\ X=\frac{4}{3} \end{bmatrix}$$

6. Calculator: 
$$6x^2 - 11x - 10$$



### Lesson #68b Homework: Solving Quadratic Functions

#### Solve by any method: 1. $x^2 + 8x = 9$

1. 
$$x^{2}+8x=9$$
  
 $X^{2}+8x-9=0$   
 $(x+9)(x-1)=0$   
 $x+9=0$   $x-1=0$   
 $X=-9$   $x=1$ 

2. 
$$2x^{2}+19x=-24$$
 $2x^{2}+19x+24=0$ 
 $(x+16)(x+3)=0$ 
 $(x+8)(2x+3)=0$ 
 $x+8=0$ 
 $x+3=0$ 
 $x=-8$ 
 $x=-3$ 
 $x=-3$ 
 $x=-3$ 

3. 
$$x^2 = 25$$
  
 $X = \sqrt{25}$   
 $X = \pm 5$ 

4. 
$$4x^{2} - 49 = 0$$

$$4x^{2} - 49$$

$$4$$

$$x^{2} - 49$$

$$4$$

$$x^{2} - 49$$

$$4$$

$$x^{3} - 49$$

$$4$$

$$4$$

#### Lesson #69 Solving Quadratic Equations Using the Quadratic Formula

Success Criteria: I can identify the a, b and c values in a quadratic equation. I can use the quadratic formula to solve a quadratic equation.

The Quadratic Formula: (Useful when the trinomial does not factor)

For any quadratic equation  $ax^2 + bx + c = 0$ , the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
 We can read this as: "x equals the opposite of b, plus or minus the square root of b squared minus 4ac, all divided by 2a"

Example #1: Use the Quadratic Formula to solve the following equations.

Solution:  
First pick out 
$$a = 4$$
,  $b = -13$ ,  $c = 3$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
First pick out  $a = -5$ ,  $b = 1$ ,  $c = 4$ 

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{13 \pm \sqrt{169 - 48}}{8}$$

$$x = \frac{13 \pm \sqrt{121}}{8}$$

$$x = \frac{13 \pm 11}{8}$$

$$x = \frac{13 \pm 11}{8}$$
or  $x = \frac{13 \pm 11}{8}$ 

$$x = \frac{24}{8}$$
 or  $x = \frac{24}{8}$  or  $x = \frac{2}{8}$ 

$$x = 3$$
 or  $x = \frac{1}{4}$ 

(B)  $-5x^2 + x + 4 = 0$ 
First pick out  $a = -5$ ,  $b = 1$ ,  $c = 4$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-5)(4)}}{2(-5)}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-5)(-5)}}{2(-5)}$$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(-5)(-5)}}{2(-5)}$$

(C) 
$$-x^{2}-1=-6x$$
  
 $+x^{2}+1$   $+x^{2}+1$   
(Note: First put it in standard form)  
 $0=x^{2}-(6x+1)$   
 $0=1$   $b=-(6)$   $c=1$   
 $x=\frac{(6\pm\sqrt{3})^{2}-4(1)(1)}{2}$   
 $x=\frac{(6\pm\sqrt{3})^{2}}{2}$   
 $x=\frac{(6\pm\sqrt{3})^{2}}{2}$   
 $x=\frac{(6\pm\sqrt{3})^{2}}{2}$   
 $x=\frac{(6\pm\sqrt{3})^{2}}{2}$   
 $x=\frac{(6\pm\sqrt{3})^{2}}{2}$ 

(D) 
$$-4x+1=-4x^{2}$$
  
 $4x^{2}-4x+1=0$   
 $X=\frac{4\pm\sqrt{1-4}^{2}-4(4)(1)}{2(4)}$   
 $X=\frac{4\pm\sqrt{10}-10}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$   
 $X=\frac{4}{8}$   
 $X=\frac{4\pm\sqrt{0}}{8}$ 

Example #3: The average monthly bill y (in dollars) for a customer's cell phone x years after the year 2000 can be modeled by  $y = -0.2x^2 + 2x + 45$ . In what year was the average monthly bill about \$50?

0.34 = 0.17

$$\begin{array}{c|c}
-2 & \sqrt{4-4} \\
\hline
-0.4 \\
X = -2 & \sqrt{10} \\
\hline
-0.4 \\
X = -2 & -0.4 \\
X = -2 & -0.4 \\
X = -5
\end{array}$$

#### Lesson #69b Quadratic Formula Practice

Success Criteria: I can identify the a, b and c values in a quadratic equation. I can use the quadratic formula to solve a quadratic equation.

The Quadratic Formula: For any quadratic equation  $ax^2 + bx + c = 0$ , the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 1. Set equation equal to zero.
- 2. Identify a, b, c.
- 3. Write the quadratic formula.
- 4. Substitute values.
- 5. Simplify (calculator).

Example #1: Use the Quadratic Formula to solve the following equations.

(A) 
$$2x^2 - 4x - 6 = 0$$

Pick out 
$$a=2$$
,  $b=4$ ,  $c=-6$ 

Write: 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16+48}}{4}$$
Simplify:  $x = \frac{4 \pm \sqrt{64}}{4}$   $\frac{4 \pm 8}{4}$   $\frac{12}{4}$ 

$$x = 3 \text{ or } x = -1$$

**(B)** 
$$2x^2 + 5x - 12 = 0$$

Follow the same sequence of steps as (A).

$$Q=2 b=5 C=-12$$

$$X = -5 \pm \sqrt{5^2 - 4(2)(-12)}$$

$$X = -5 \pm \sqrt{25 + 9L}$$

$$X = -5 \pm \sqrt{121}$$

$$4$$

$$X = -5 \pm 11$$

$$4$$

$$-5 \pm 11$$

$$4$$

$$-5 \pm 11$$

(C)Graph part (B) on the calculator and find both zeros.

$$x = 1.5$$
 or  $x = -4$ 

**(D)** Solve 
$$2x^2 + 3x = 8$$

(Note: First put it in standard form)
$$2 \times^{2} + 3 \times -8 = 0$$

$$X = -3 \pm \sqrt{3^{2} - 4(2)(-8)}$$

$$2(2)$$

$$X = -3 \pm \sqrt{9 + 64}$$

$$4$$

$$X = -3 \pm \sqrt{73}$$

$$2$$

$$X = -3 \pm 8.54$$

$$\frac{1}{2} = \frac{3 + 8.54}{2}$$

$$\frac{-3 + 8.54}{2} = \frac{-3 - 8.54}{2}$$

$$\frac{5.54}{2} = 2.77$$
  $\frac{-11.54}{2} = \frac{-5.77}{2}$ 

(E) Solve 
$$-10x + 6x^2 = -5$$

$$6x^2-10x+5=0$$

$$X = \frac{10 \pm \sqrt{(-10)^2 - 4(6)(5)}}{2(6)}$$

$$X = \frac{10 \pm \sqrt{-20}}{12}$$

no Solution

(F) Calculate  $b^2 - 4ac$  from (E). How could that help you determine the answer?

$$(-10)^2 - 4(6)(5)$$
  
 $100 - 120$ 

negative

Example #2: The number y of Northern Rocky Mountain wolf breeding pairs x years since 1995 can be modeled by  $y = 0.34x^2 + 3.0x + 9$ . When were there about 30 breeding pairs?

$$36 = 0.34 \times^2 + 3 \times +9$$

$$X = \frac{-3 \pm \sqrt{3^2 - 4(0.34)(-21)}}{2(0.34)}$$

$$\frac{-3-6.13}{0.68} = \frac{-9.13}{0.68} = -13.4$$

# Lesson #70 Choosing a Solution Method

Success Criteria: I can solve quadratic equations by undoing operations, factoring the quadratic formula or the calculator. I can determine what method is best depending on the problem.

Solve  $x^2 - 3 = 0$  by different methods:

a) Calculator

$$X = \pm 1.73$$

c) UNDOing operations

See Concept Map

b) Factoring
not factorable

Example #1: For each equation, decide on what method (other than by graphing on a calculator) might be easiest to use to solve it, and then solve the equation.

**(A)** 
$$x^2 - 10 = 0$$

Which method appears to be easiest?

$$\chi^2 = 10$$
  
 $\chi = \sqrt{10}$ 

**(B)** 
$$x^2 - 10x = 0$$

Which method appears to be easiest?

(C) 
$$x^2 + 8x + 12 = 0$$

Which method appears to be easiest?

Why?

Why? 
$$a=1$$
  $b=8$   $c=12$ 

**(D)** 
$$4x^2 = 36$$

Which method appears to be easiest?

# Undoing operations

Why?

$$\frac{4x^2}{4} = \frac{36}{4}$$

$$x^2 = 9$$

(E) 
$$x^2 + 2 = 5x$$
  
 $x^2 - 5x + 2 = 0$ 

Which method appears to be easiest?

Calculator / Quad Form

Why?

$$X = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$X = \frac{5 \pm \sqrt{25-8}}{2}$$

$$X = \frac{5 \pm \sqrt{17}}{2}$$

$$X = \frac{5 \pm 4.12}{2} = \frac{9.12}{2} = 4.56$$

$$X = \frac{5 \pm 4.12}{2} = 0.88 = 0.44$$

(F) 
$$4x^2 = 7x - 10$$
  
 $4x^2 - 7x + 10 = 0$ 

Which method appears to be easiest?

Calculator/Quad Form

Why?

# Lesson #70b

**Solving Equation Practice** 

Success Criteria: I can determine how many zeros an equation has by using my calculator. I can solve quadratic equations with a success Criteria: I can determine how many zeros an equation has by using my calculator. I can solve quadratic equations with the guadratic model. equations using many different methods. I can answer questions about a real life quadratic model.

Use your graphing calculator to determine HOW MANY solutions the equation will have (0, 1, or 2). Then find the solutions using the given method.

1.  $x^2 + 8x - 20$  by Factoring (x+10)(x-2)



Number of Solutions: 2

Solutions: <u>-10, 2</u>

2.  $x^2 = -4$  by UNDOing operations  $X = \sqrt{-4}$ 

Number of Solutions: O

no solution

Solutions: None

3.  $13x = 2x^2 + 6$  by Quadratic Formula Number of Solutions: 2

 $0=2x^2-13x+6=1$  0=2 b=-13 c=6 Solutions:  $6,\frac{1}{2}$ 

 $X = \frac{13 \pm \sqrt{(-13)^2 - 4(2)(6)}}{2(2)}$   $X = 13 \pm \sqrt{169 - 48}$   $13411 = \frac{24}{4} = 6$ 

- $X = \frac{13 \pm \sqrt{121}}{4}$   $X = \frac{13 \pm 11}{4}$   $X = \frac{13 11}{4} = \frac{2}{4} = \frac{1}{2}$

What other methods could have been used other than Quadratic Formula on #3?

calculator Factoring

4.  $x^2 = 8$  by UNDOing operations x= 18

Number of Solutions: 2

X= +2.83

Solutions:  $\pm 2.83$ 

5. 
$$9-24x = -16x^2$$
 by Graphing  $+16x^2 + 16x^2$  by Graphing  $16x^2 - 24x + 9 = 0$ 

Number of Solutions: 1

Solutions: 0.75

6. 
$$2x-9=14$$
 by UNDOing operations  $\frac{2x=19}{2}$   $\frac{2}{2}$   $x=9.5$ 

Number of Solutions: 1

Solutions: 9.5

Solve the remaining problems by any appropriate method.

7. 
$$2x^{2} + 8x = 0$$
  
 $2x(x+4)=0$   
 $2x=0$   $x+4=0$   
 $-4-4$   
 $5x=4$   
 $5x=4$   
 $5x=4$   
 $5x=4$   
 $5x=4$ 

8. 
$$x^{2} - 4x = -4$$
  
 $x^{2} - 4x + 4 = 0$   
 $(x-2)(x-2) = 0$   
 $x-2=0$   
 $x-2=0$   
 $x-2=0$   
 $x=2$   
 $x=2$   
 $x=3$   
 $x=4$ 

11. The height h (in feet) of a javelin thrown at a track and field competition can be modeled by  $h = -16t^2 + 50t + 6$ , where t is time in seconds.

a. What is the highest the javelin gets?

b. How high is the javelin after 1 second?

c. After how many seconds is the javelin 30 feet above the ground?