# Lesson #48
# Polynomials

Success Criteria: I can determine if an expression is a polynomial. I can write a polynomial in standard form. I can determine the degree and leading coefficient of a polynomial.

**Definitions and Examples:**

**Polynomial:** A Polynomial Expression is the sum of terms in the form $\mathbf{ax^k}$ (k is a nonnegative integer)

*Example:* $3x^2 - 5 - 2x^3 + 6x$

*Polynomials cannot have exponents that are fractions/decimals, negative, or variables.

**Standard form of a Polynomial:** Terms are placed in descending order by degree (exponents).

*Example:* $-2x^3 + 3x^2 + 6x - 5$

**Degree of Polynomial:** Largest degree (exponent) of its terms

Degree: 3

**Leading Coefficient:** Coefficient (number in front) of the first Term when in Standard Form

L.C.: -2

**Example 1: Decide whether the given expression is a polynomial. If it is not a polynomial, explain.**

a) $3^x$ not polynomial exponent can't be a variable

b) $x^{\frac{1}{2}}$ not polynomial exponent can't be a fraction

c) $-2x^3 + 5x - 2$ polynomial

**Example 2: Rewrite the following expressions in standard form. Identify the degree and the leading coefficient.**

a) $5y - 2y^2 + 9$

$-2y^2 + 5y + 9$

Degree: 2

Leading coef: -2

b) $1 - 3b^2 + 4b^4 - 6b^5 + b^3$

$-6b^5 + 4b^4 + b^3 - 3b^2 + 1$

Degree: 5

Leading coef: -6

Success Criteria: I can classify a polynomial by the number of terms.

**Classifying Polynomials by Number of Terms:**

**Monomial**: A polynomial with one term. *Examples*: $-4$ or $\frac{1}{2}y^2$

**Binomial**: A polynomial with two terms. *Example*: $6x^3 - 5x$

**Trinomial**: A polynomial with three terms. *Example*: $5x^2 + x - 4$

**Polynomial**: Used for any polynomial expression with 4 or more terms. *(Although Polynomial is a cover-all name for monomials, binomials, and trinomials too.) Example:* $x^4 + 3x^2 - x + 6$

**Example 3: Classifying Polynomials by its degree and number of terms.**
***Be sure the expressions are in standard form first***

| Polynomial | Degree | Leading Coefficient | Number of Terms | Classify by Number of Terms |
|---|---|---|---|---|
| 8 | 0 | 8 | 1 | monomial |
| $-12x^6$ | 6 | -12 | 1 | monomial |
| $2+x^2-5x$ | 2 | 1 | 3 | trinomial |
| $5x^3-7$ | 3 | 5 | 2 | binomal |
| $-7x^3+5x-2x^7+4$ | 7 | -2 | 4 | polynomial |

Success Criteria: I can use a polynomial to determine the height of an object after a certain amount of time.

**Vertical Movement Applications:**

The polynomial $-16t^2 + v_0 t + s_0$ represents the HEIGHT of an object, where

$s_0$ = the initial (or starting) height in *feet*

$v_0$ = the initial (or starting) velocity in *feet per second*

$t$ = the time in *seconds*

Note: If the object initially moves **upward**, then $v_0$ is **positive**.

If the object initially moves **downward**, then $v_0$ is **negative**.

**Example 4:** A baseball player throws a baseball upward into the air with a velocity of 30 feet per second. The ball is 5 feet above the ground when it leaves the player's hand.

| (A) Write a polynomial that represents the height of the baseball. | (B) Find the height of the baseball after 1 second. | (C) Find the height of the baseball after 2 seconds. |
|---|---|---|
| $h = -16t^2 + 30t + 5$ | $h = -16(1)^2 + 30(1) + 5$<br>$h = -16 + 30 + 5$<br>$h = 19$ ft | $h = -16(2)^2 + 30(2) + 5$<br>$h = -16 \cdot 4 + 60 + 5$<br>$h = -64 + 60 + 5$<br>$h = 1$ ft |

# Lesson #48
## On Your Own

**Decide whether the given expression is a polynomial. If it is not a polynomial, explain.**

1. $y^{-1}$ No - exponent can't be negative

2. $6+5x^3$ Polynomial

3. $-55$ polynomial

4. $5h^{-3}+4h^2+2$ No - exponents can't be negative

**Rewrite the following expressions in standard form. Identify the degree and the leading coefficient.**

5. $6-2x^5-4x^3$ $-2x^5-4x^3+6$ Degree: 5 Leading Coef: -2

6. $-2r+5r^3-6$ $5r^3-2r-6$ Degree: 3 Leading Coef: -2

**Classifying the polynomial by its number of terms. (What do you have to make sure of first??)**

7. $14w^3-9w^2$ Binomial

8. $7-3x+12x^4$ Trinomial

9. $8+5y^2-3y+2y^5-y^9$ Polynomial

10. $5d-3d^2+8$ Trinomial

11. You are playing angry birds and you want to calculate the location of your bird at different periods of time. You shoot the bird from a height of 50 feet above the ground. The initial velocity of the bird is 15 feet per second.

a. Write the equation that models this situation.

$v(t)=-16t^2+15t+50$

b. What is the height of your bird after 2 seconds

$v(2)=-16(2)^2+15(2)+50$

$=-16\cdot 4+30+50$

$=-64+30+50$

$=\boxed{16 \text{ ft}}$

# Lesson #49
## Adding and Subtracting Polynomials

Success Criteria: I can combine like terms in polynomials. I can set up polynomials to represent the area of a figure.

**Adding and Subtracting Polynomials:**

*Always remember to **add the opposite** first when subtracting!!!

**Examples: Add or subtract each polynomial expression. Write your answer in standard form. Then classify it as a monomial, binomial, trinomial, or just a polynomial.**

1. $(2x^3-5x^2+x)+(2x^2+x^3-1)$

$3x^3-3x^2+x-1$

Polynomial

2. $(3x^2+x-6)+(x^2+4x)$

$4x^2+5x-6$

Trinomial

3. $(4x^2-13x)+(-5x^2-7)$

$-x^2-13x-7$

Trinomial

4. $(4x^2-3x+5)-(3x^2-x-8)$

$4x^2-3x+5-3x^2+x+8$

$x^2-2x+13$

Trinomial

5. $(4x^2+5)-(-2x^2+2x-4)$

$4x^2+5+2x^2-2x+4$

$6x^2-2x+9$

Trinomial

6. $(5x^4-4x+1)-(8-x^4)$

$5x^4-4x+1-8+x^4$

$6x^4-4x-7$

Trinomial

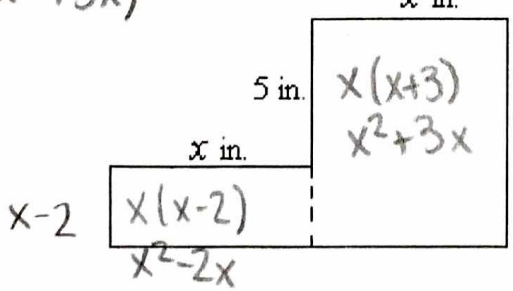
EXAMPLE 7: Use the diagram shown below.

**(A)** Write a polynomial that represents the total area of the figure.

$(x^2-2x)+(x^2+3x)$

$2x^2+x$

**(B)** Find the total area of the figure when $x = 4$ inches.




$2(4)^2+4$

$2\cdot16+4$

$32+4 = 36$ in

$x+3-5$

$x-2$

**PRACTICE: Add or subtract each polynomial expression. Write your answer in standard form. Then classify it as a monomial, binomial, trinomial, or just a polynomial.**

1. $(6x - x^2 + 3) + (4x^2 - x - 2)$

$3x^2 + 5x + 1$

Trinomial

2. $(8x - 9x^4 + 2x^3) + (1 - x - 6x^3)$

$-9x^4 - 4x^3 + 7x + 1$

Polynomial

3. $(3x^2 - 5x + 3) - (2x^2 - x - 15)$

$3x^2 - 5x + 3 - 2x^2 + x + 15$

$x^2 - 4x + 18$

Trinomial

4. $(x^5 + 7x^3) - (1 + 5x^4 - x^3)$

$x^5 + 7x^3 - 1 - 5x^4 + x^3$

$x^5 - 5x^4 + 8x^3 - 1$

Polynomial

Sum & Product Puzzle 1

# Lesson #49b
# Intro – Multiplying Polynomials Using a Table

Success Criteria: I can multiply binomials using a table.

Multiply the binomials $(x + 5)(x + 4)$ using a table.

$x^2+5x+4x+20$

$x^2+9x+20$

| | x | 4 |
|---|---|---|
| x | $x^2$ | $4x$ |
| 5 | $5x$ | 20 |

Multiply the binomials $(x + 3)(x - 2)$ using a table.

$x^2+3x-2x-6$

$x^2+x-6$

| | x | -2 |
|---|---|---|
| x | $x^2$ | $-2x$ |
| 3 | $3x$ | $-6$ |

Multiply the binomials $(x - 8)(x - 1)$ using a table.

$x^2-8x-x+8$

| | x | -1 |
|---|---|---|
| x | $x^2$ | $-x$ |
| -8 | $-8x$ | $+8$ |

# Lesson #49b
# Lesson 48 – 49 Review

Adding and Subtracting Polynomial Jeopardy on whiteboards:

Groups of 2 – each question you rotate who writes.

# Lesson #50
# Multiplying Polynomials

Success Criteria: I can multiply two terms. I can distribute a term to a polynomial.

**Example 1: Multiplication Review:**

1. $-2x \cdot -4x$

$8x^2$

2. $5x^2 \cdot 3x$

$15x^3$

**Example 2: Distributive Property**

1. $-3a(a^2 - 6a + 17)$

$-3a^3 + 18a^2 - 51a$

2. $9st(3s + 4t - 3s^2)$

$27s^2t + 36st^2 - 27s^3t$

**You Try! Multiply**

1. $-4t^3(t^2 + 7t - 5)$

$-4t^5 - 28t^4 + 20t^3$

2. $7mr(12m - 7r + 5)$

$84m^2r - 49mr^2 + 35mr$

Success Criteria: I can use the box or foil method to simplify a polynomial.

**Example 3: Multiplying Binomials**

| | Foil Method (Distributive Property) | Dist. Prop. organized in a table |
|---|---|---|
| 1. $(4x-1)(3x+8)$ | First Outer Inside Last<br>$4x \cdot 3x$ $4x \cdot 8$ $-1 \cdot 3x$ $-1 \cdot 8$<br>$12x^2$ $32x$ $-3x$ $-8$<br>$12x^2 + 29x - 8$ | <table><tr><td></td><td>3x</td><td>8</td></tr><tr><td>4x</td><td>$12x^2$</td><td>32x</td></tr><tr><td>-1</td><td>-3x</td><td>-8</td></tr></table><br>$12x^2 - 3x + 32x - 8$<br>$12x^2 + 29x - 8$ |

| | Foil Method (Distributive Property) | Dist. Prop. organized in a table |
|---|---|---|
| 2. $(b+8)(6-2b^2)$ | F: $b \cdot 6 = 6b$<br>O: $b \cdot -2b^2 = -2b^3$<br>I: $8 \cdot 6 = 48$<br>L: $8 \cdot -2b^2 = -16b^2$<br>$-2b^3 - 16b^2 + 6b + 48$ | <table><tr><td></td><td>6</td><td>$-2b^2$</td></tr><tr><td>b</td><td>6b</td><td>$-2b^3$</td></tr><tr><td>8</td><td>48</td><td>$-16b^2$</td></tr></table><br>$-2b^3 - 16b^2 + 6b + 48$ |
| 3. $(5-2x)^2$<br>$(5-2x)(5-2x)$ | F: $5 \cdot 5 = 25$<br>O: $5 \cdot -2x = -10x$<br>I: $-2x \cdot 5 = -10x$<br>L: $-2x \cdot -2x = 4x^2$<br>$4x^2 - 10x - 10x + 25$<br>$4x^2 - 20x + 25$ | <table><tr><td></td><td>5</td><td>$-2x$</td></tr><tr><td>5</td><td>25</td><td>$-10x$</td></tr><tr><td>$-2x$</td><td>$-10x$</td><td>$4x^2$</td></tr></table><br>$4x^2 - 20x + 25$ |

**Practice! Multiply.**

1. $(b-8)(5b-2)$

$5b^2 - 2b - 40b + 16$

$5b^2 - 42b + 16$

2. $(-3d+10)(2d-1)$

$-6d^2 + 3d + 20d - 10$

$-6d^2 + 23d - 10$

Sum + Product #2

# Lesson #51
# Multiplying Polynomials

Success Criteria: I can multiply two binomials.

**Example 1: Multiplying Polynomials**

**Distributive Property**

1. $(2x-3)(x+7)$

$2x(x+7)-3(x+7)$

$2x^2+14x-3x-21$

$2x^2+11x-21$

**Table**

| | $x$ | $7$ | |
|---|---|---|---|
| $2x$ | $2x^2$ | $14x$ | |
| $-3$ | $-3x$ | $-21$ | |

$2x^2+11x-21$

2. $(x+3)(-2x^2+x+4)$

$x(-2x^2+x+4)+3(-2x^2+x+4)$

$-2x^3+x^2+4x-6x^2+3x+12$

$-2x^3-5x^2+7x+12$

| | $-2x^2$ | $x$ | $4$ |
|---|---|---|---|
| $x$ | $-2x^3$ | $x^2$ | $4x$ |
| $3$ | $-6x^2$ | $3x$ | $12$ |

$-2x^3-6x^2+x^2+3x+4x+12$

~~$-2x^3-5x^2+7x+12$~~

3. $(a+4)(a^2-2a+3)$

$a(a^2-2a+3)+4(a^2-2a+3)$

$a^3-2a^2+3a+4a^2-8a+12$

$a^3+2a^2-5a+12$

| | $a^2$ | $-2a$ | $3$ |
|---|---|---|---|
| $a$ | $a^3$ | $-2a^2$ | $3a$ |
| $4$ | $4a^2$ | $-8a$ | $12$ |

$a^3+4a^2-2a^2-8a+3a+12$

$a^3+2a^2-5a+12$

**Practice! Multiply.**

1. $(3-6a)(4a-1)$

$12a-3-24a^2+6a$

$-24a^2+18a-3$

2. $(3x-1)^2$

$(3x-1)(3x-1)$

$9x^2-3x-3x+1$

$9x^2-6x+1$

3. $(2s+5)(s^2+3s-1)$

$2s(s^2+3s-1)+5(s^2+3s-1)$

$2s^3+6s^2-2s+5s^2+15s-5$

$2s^3+11s^2+13s-5$

4. $(w-3)(2w^2+8w+1)$

$w(2w^2+8w+1)-3(2w^2+8w+1)$

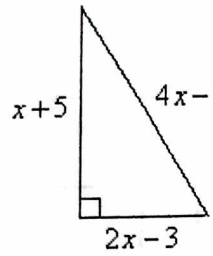
$2w^3+8w^2+w-6w^2-24w-3$

$2w^3+2w^2-23w-3$

Success Criteria: I can create polynomials to represent a situation.

Example #2 Write a polynomial for the area of each shape.

**(A)**



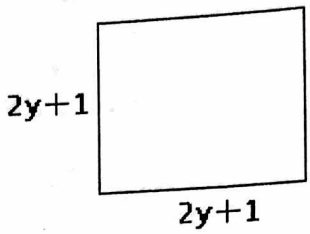

$A = \frac{1}{2}(2x-3)(x+5)$

$A = (x-\frac{3}{2})(x+5)$

$= x^2 + 5x - \frac{3}{2}x - \frac{15}{2}$

$= x^2 + \frac{7}{2}x - \frac{15}{2}$

**(B)**

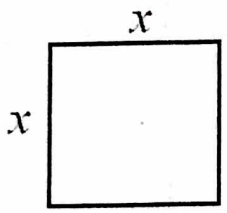


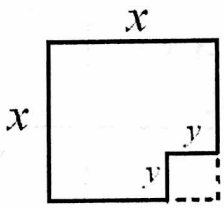

$A = (2y+1)(2y+1)$

$A = 4y^2 + 2y + 2y + 1$

$A = 4y^2 + 4y + 1$

Example #3 Suppose you start with a square piece of paper that is $x$ inches by $x$ inches. You then cut out a square that is $y$ inches by $y$ inches from one corner.

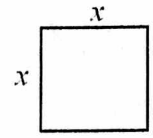



We now want to find an expression for the area of the remaining paper. There are several methods we can use to find this area.

Method 1:

*Take the area of*

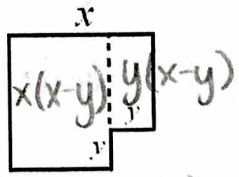



*and subtract the area of*


$x^2 - y^2$

Method 2:

Separate the paper into two rectangles, then find their areas and add them together.




$x(x-y) + y(x-y)$

$x^2 - xy + xy - y^2$

$x^2 - y^2$

Sum & Product #3

# Lesson #52
# Special Products of Polynomials

Complete "Record and Practice Journal" p. 188 #1-2 using a table or FOIL.
What do you notice?

Complete "Record and Practice Journal" p. 188 #5-6 using a table or FOIL.
What do you notice?

Success Criteria: I can multiply polynomials using any method. I can identify patterns when multiplying polynomials.

- **Sum & Difference Pattern:** $(a + b)(a - b) =$ $a^2 - b^2$

**Example 1: Sum and Difference Pattern**
**Distributive Property**

1. $(x-7)(x+7)$

$x(x+7) - 7(x+7)$

$x^2 + 7x - 7x - 49$

$x^2 - 49$

**Table**

| | $x$ | $7$ | |
|---|---|---|---|
| $x$ | $x^2$ | $7x$ | |
| $-7$ | $-7x$ | $-49$ | |

$x^2 - 49$

2. $(6+x)(6-x)$

$6^2 - x^2$

$36 - x^2$

3. $(3x-4)(3x+4)$

$(3x)^2 - 4^2$

$9x^2 - 16$

- **Square of a Binomial Pattern:** $(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

**Example 1: Square of a Binomial Pattern**

**Distributive Property (FOIL)**

1. $(x+7)^2$

$x^2+2\cdot7x+7^2$

$x^2+14x+49$

**Table**

| | x | 7 | |
|---|---|---|---|
| x | $x^2$ | $7x$ | |
| 7 | $7x$ | 49 | |

$x^2+7x+7x+49 = x^2+14x+49$

2. $(y-3)^2$

$y^2-2\cdot y\cdot3+3^2$

$y^2-6y+9$

3. $(4+3x)^2$

$4^2+2\cdot4\cdot3x+(3x)^2$

$16+24x+9x^2$

**Practice: Simplify.**

1. $(x+5)(x-5)$

$x^2-5^2$

$x^2-25$

2. $(x-9)^2$ $(x-9)(x-9)$

$x^2-2\cdot9\cdot x+9^2$

$x^2-18x+81$

3. $(6+7x)^2$

$6^2+2\cdot7x\cdot6+(7x)^2$

$36+84x+49x^2$

4. $(3x-8)(3x+8)$

$(3x)^2-8^2$

$9x^2-64$