Lesson #61
Graphing $y = ax^2$

Success Criteria: I can graph a quadratic equation from a table. I can determine what a graph will look like with a negative a value. I can determine what a graph will look like when you have an a value bigger than one or between zero and one.

Problem #1:

(A) For each equation, make a table of ordered pair solutions, then graph the equation.

Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

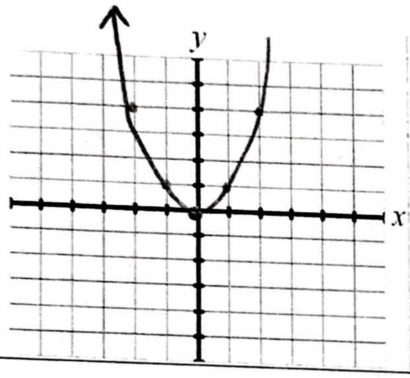
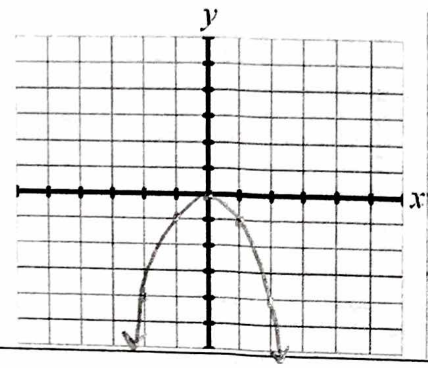


Table and Graph of $y = -x^2$

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4



(B) Complete the following statement:

Statement #1: "When the x^2 in the equation $y = x^2$ is multiplied by a NEGATIVE to form the equation $y = -x^2$, the graph flips upside down."

Statement #2: "The reason this happens is because you are taking the y-values for the equation $y = x^2$ and multiplying all of them by a -1 when getting the y-values for the equation $y = -x^2$.

(C) Which graph "opens up"? $y=x^2$

Which graph "opens down"? $y=-x^2$

Which graph has the vertex (highest/lowest point) at the top of the curve? $y=-x^2$

Which graph has the vertex (highest/lowest point) at the bottom of the curve? $y=x^2$

What are the coordinates of the vertex? (0, 0)

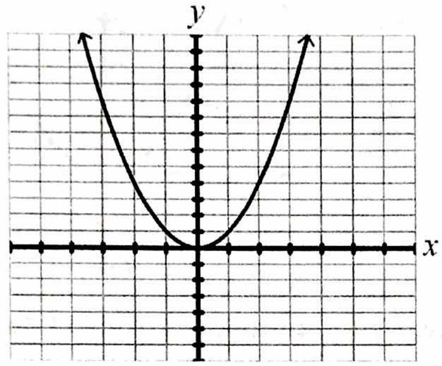
Problem #2:

Let's now investigate the effect of multiplying the x^2 in the equation $y = x^2$ by a number a which is greater than 1. In other words, how does the a in the equation $y = ax^2$ affect the graph of the equation $y = x^2$ when $a > 1$?

(A) A table and the graph of $y = x^2$ is given below.

Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



For each given equation below, make a table of ordered pair solutions, then plot and connect the solution points to graph the equation.

Table and Graph of $y = 2x^2$

x	y
-2	8
-1	2
0	0
1	2
2	8

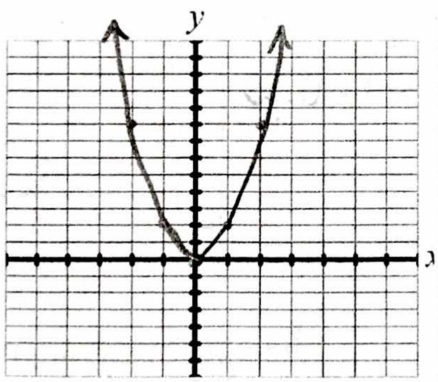
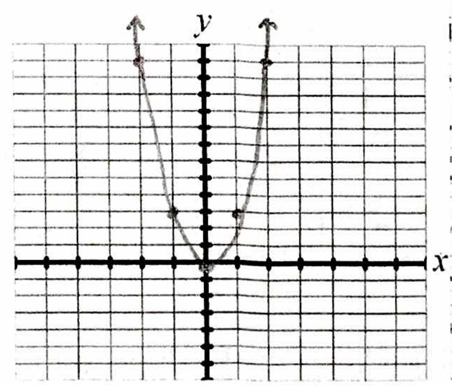


Table and Graph of $y = 3x^2$

x	y
-2	12
-1	3
0	0
1	3
2	12



(B) Compare the three graphs above, then complete the following observations/statements:

Statement #1: "When the x^2 in the equation $y = x^2$ is multiplied by a number $a > 1$, the graph of the equation $y = ax^2$ will be thinner than the graph of $y = x^2$."

taller narrower

Statement #2: "The reason this happens is because the previous ordered pair y-values for the equation $y = x^2$ are now "a" times bigger than before, making the graph rise more quickly.

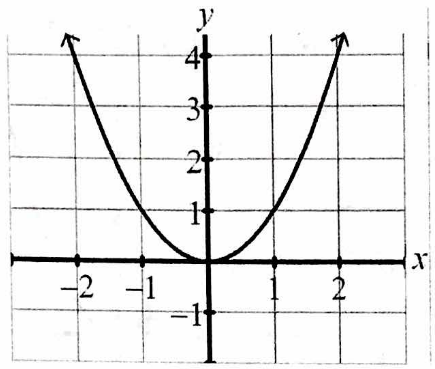
Statement #3: "The larger we make the value of a, the narrower the graph will be."

Problem #3:

Let's now investigate the effect of multiplying the x^2 in the equation $y = x^2$ by a number a which is BETWEEN 0 and 1. In other words, how does the a in the equation $y = ax^2$ affect the graph of the equation $y = x^2$ when $0 < a < 1$?

(A) Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



For each given equation below, make a table of ordered pair solutions, then plot and connect the solution points to graph the equation.

Table and Graph of $y = \frac{1}{2}x^2$

x	y
-2	2
-1	0.5
0	0
1	0.5
2	2

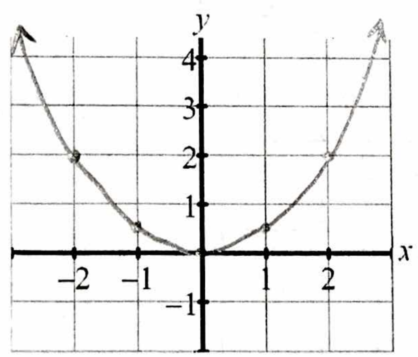
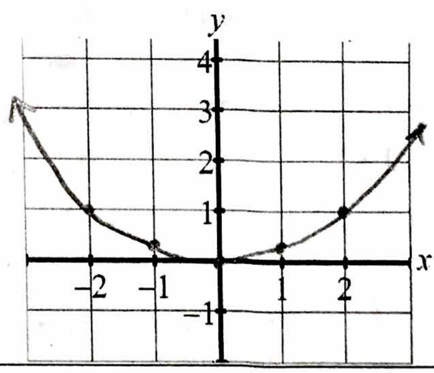


Table and Graph of $y = \frac{1}{4}x^2$

x	y
-2	1
-1	0.25
0	0
1	0.25
2	1



(B) Compare the three graphs above, then complete the following observations/statements:

Statement #1: "When a is a number BETWEEN 0 and 1, in other words $0 < a < 1$, the graph of the equation $y = ax^2$ will be wider "Shorter" than the graph of $y = x^2$."

Statement #2: "The smaller we make the value of a for $0 < a < 1$, the wider the graph will be."

(C) Explain why statements #1 and #2 are true. The values are multiplied by whatever a is

Homework: Worksheet 8.1 Puzzle Time

Lesson #62
Graphing $y = ax^2 + c$

Success Criteria: I can determine what will happen to a graph when you add or subtract a number. I can determine the domain and range of a quadratic graph.

Problem #1:

(A) Graph $y = x^2$ and $y = x^2 + 2$.

Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

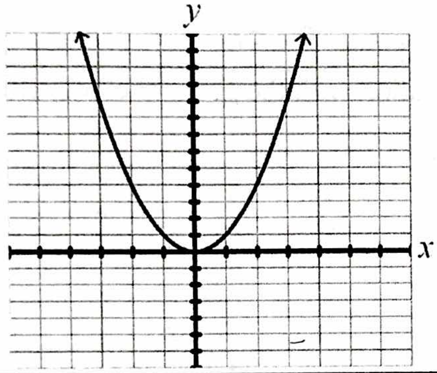
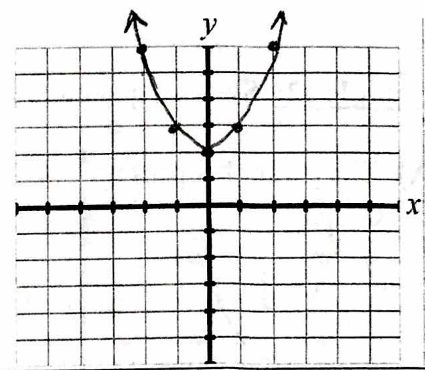


Table and Graph of $y = x^2 + 2$

x	y
-2	6
-1	3
0	2
1	3
2	6



(B) Compare the graph of $y = x^2 + 2$ to the graph of $y = x^2$.

Moved up 2 units

(C) Use the graph to find the Domain and the Range for the equation.

Domain: $\mathbb{R}$ ____________ Range: $y \geq 2$ ____________

x-values y-values

(D) Graph $y = x^2$ and $y = x^2 - 3$.

Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

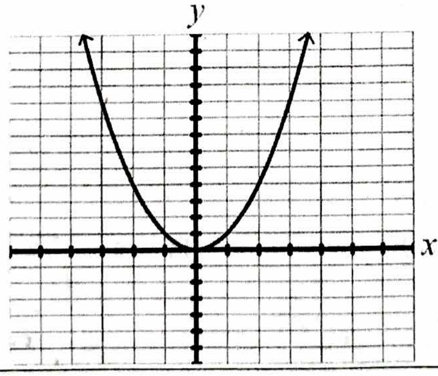
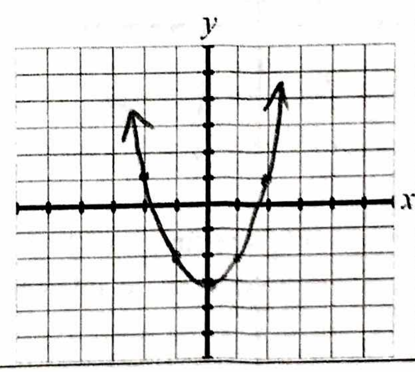


Table and Graph of $y = x^2 - 3$

x	y
-2	1
-1	-2
0	-3
1	-2
2	1



(E) Compare the graph of $y = x^2 - 3$ to the graph of $y = x^2$.

Moved down 3 units

(F) Use the graph to find the Domain and the Range for the equation.

Domain: $\mathbb{R}$ ____________ Range: $y \geq -3$ ____________

Success Criteria: I can define key features of a quadratic graph. I can find the key features of a parabola. I can write an equation from a graph.

PARABOLA: The U-shaped graph of $y = ax^2 + c$.

VERTEX : The highest or lowest point on the parabola. 

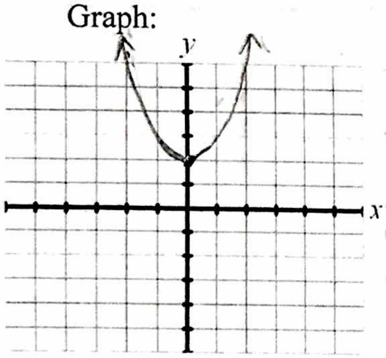
AXIS of SYMMETRY : The line that divides the parabola into two symmetric pieces. 

ZEROs: The x-values for which $y = 0$ (the x-intercepts). 

Example #2: Sketch a quadratic function and write an equation with the given characteristics:

(A) The parabola opens up and the vertex is $(0,2)$.

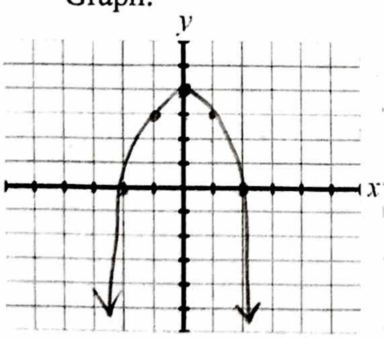
Graph:



Equation: $y = x^2 + 2$

(B) The parabola has a vertex at $(0,4)$ and has x-intercepts at $(2,0)$ and $(-2,0)$.

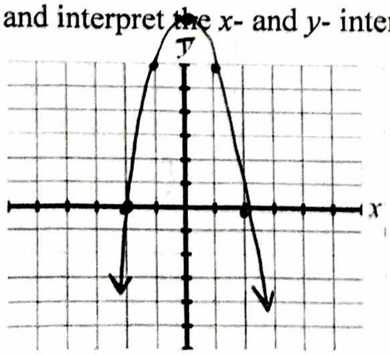
Graph:



Equation:

$y = -x^2 + 4$

Example #3: The function $y = -2x^2 + 8$ gives the height y (in feet) of an apple after falling x seconds. Find and interpret the x- and y- intercepts.



X	Y
-2	0
-1	6
0	8
1	6
2	0

x-intercepts: -2, 2

y-intercepts: 8

The apple starts at 8 feet tall & hits the ground after 2 seconds.

Example #4: Without graphing, answer the following about the graph of $y = -\frac{1}{2}x^2 + 3$.

(A) What is a? $-\frac{1}{2}$ What is c? 3

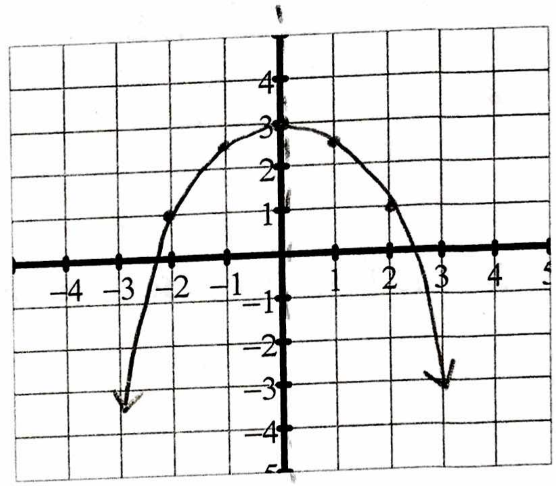
(B) Will the parabola graph be narrower or will it be wider than the graph of $y = x^2$? wider

(C) Will the parabola graph open upward Y or open downward I ? downward

(D) Will the parabola graph be shifted up or down at all? If so, how many units? up 3

(E) Graph the equation $y = -\frac{1}{2}x^2 + 3$ using a table of ordered pair solutions.

x	y
−2	1
−1	2.5
0	3
1	2.5
2	1



(F) What are the coordinates of the vertex? (0, 3)

(G) Is the vertex the Maximum point or the Minimum point of the graph? maximum

(H) Draw a dark dotted line for the axis of symmetry, then give the equation of this line: X=0

(I) Use the graph to find or estimate the zeros of the equation: $x =$ −2.2 and $x =$ 2.2

(J) Use the graph to find the Domain and the Range for the equation.

Domain: $\mathbb{R}$ Range: $y \leq 3$

Lesson #62b
Supplemental Section: Factoring and Graphing Quadratics

Success Criteria: I can use factoring to help graph a parabola. I can determine key features of a parabola using the table, equation and graph.

Example #1:

(A) Show that the equation $y = 3x^2 - 6x - 9$ **factors into** $y = 3(x-3)(x+1)$.

$3(x^2-2x-3)$

-3, 1, -3, -2

$3(x-3)(x+1)$

(B) Find the zeros of the function using the Zero-Product Property.

$x-3=0$ $\quad$ $x+1=0$

$x=3$ $\quad$ $x=-1$

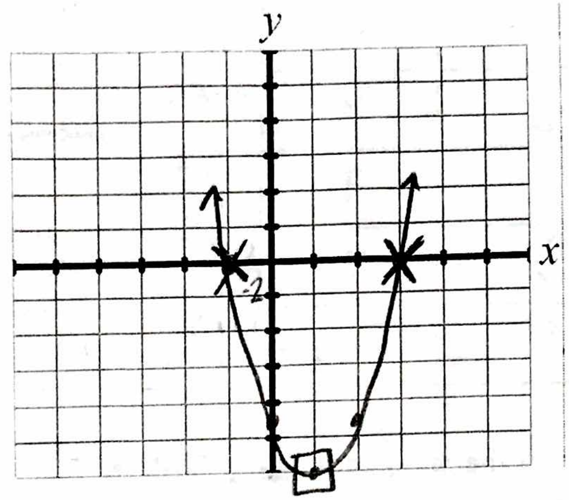
$x = 3$ and $x = -1$

(Since "zero" in math means when $y = 0$, put 0 in for y)

(C) Complete the table with points near the vertex and graph the function. Change the scale if needed.

$y = 3x^2 - 6x - 9$

x	y
-1	0
0	-9
1	-12
2	-9
3	0



(D) Mark the zeros on the graph with an X.

(E) Put a box around the vertex. What are the coordinates of the vertex? (1 , -12)

(F) What is the equation of the axis of symmetry? $x = 1$

(G) Identify the domain and range of the function.

Domain: $\mathbb{R}$ **Range:** $y \geq -12$

Example #2:

(A) Find the zeros of $y = x^2 + 2x - 8$.

$0 = x^2 + 2x - 8$

$0 = (x-2)(x+4)$

$x - 2 = 0$ $x + 4 = 0$

$x = 2$ $x = -4$

$x = -4$ and $x = 2$

(B) Will the graph open up or down? up

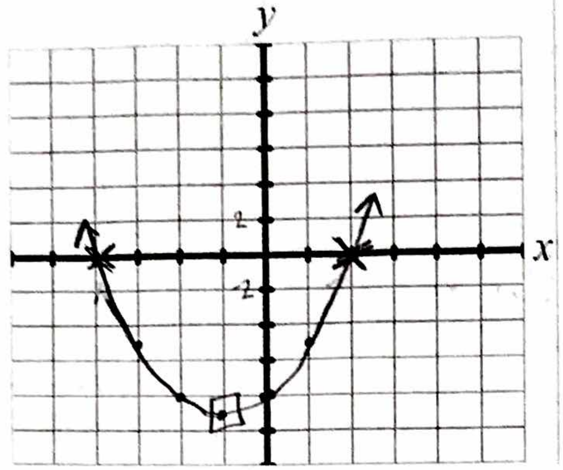
(C) Will the graph be shifted up or down compared to $y = x^2$?

down 8

(D) Complete the table with points near the vertex and graph the function. Change the scale if needed.

$y = x^2 + 2x - 8$

x	y	
-3	-5	9-6-8
-2	-8	4-4-8
-1	-9	1-2-8
0	-8	0+0-8
1	-5	1+2-8



(E) Mark the zeros on the graph with an X.

(F) Put a box around the vertex. What are the coordinates of the vertex? (-1, -9)

(G) What is the equation of the axis of symmetry? $x = -1$

(H) Circle the y-intercept. What are the coordinates of the y-intercept? (0, -8)

(I) Identify the domain and range of the function.

Domain: $\mathbb{R}$ **Range:** $y \geq -9$

Example #3:

(A) Find the zeros of $y = -2x^2 + 2$.

$-2(x^2 - 1)$

$-2(x-1)(x+1)$

$x = 1$ and $x = -1$

(B) Will the graph open up or down?

down

(C) Will the graph be narrower or wider than $y = x^2$?

narrower

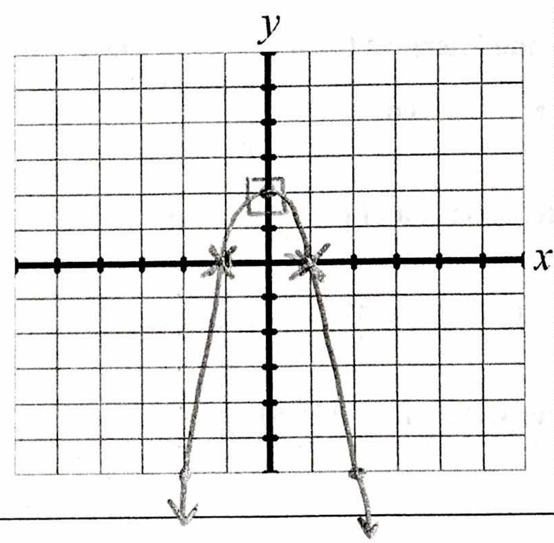
(D) Will the graph be shifted up or down compared to $y = x^2$?

up 2

(E) Complete the table with points near the vertex and graph the function. Change the scale if needed.

$y = -2x^2 + 2$

x	y	
-2	-6	-8+2
-1	0	-2+2
0	2	0+2
1	0	-2+2
2	-6	-8+2



(F) Mark the zeros on the graph with an X.

(G) Put a box around the vertex. What are the coordinates of the vertex? $(0, 2)$

(H) What is the equation of the axis of symmetry? $x = 0$

(I) Circle the y-intercept. What are the coordinates of the y-intercept? $(0, 2)$

(J) Identify the domain and range of the function.

Domain: $\mathbb{R}$ **Range:** $y \leq 2$

Example #4:

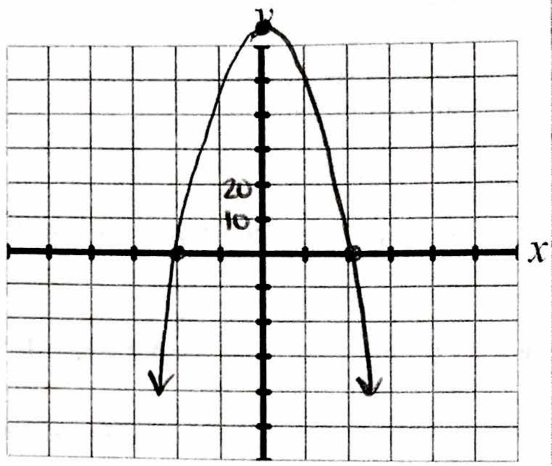
A bass swims over a waterfall. The distance y (in feet) that the fish falls in x seconds can be modeled by the equation $y = -16x^2 + 64$.

$0 = -16x^2 + 64$

$0 = -16(x^2 - 4)$

$0 = -16(x-2)(x+2)$

Bass's Distance vs. Time Graph:



(A) Find and interpret the meaning of the y-intercept and the positive x-intercept.

The y-intercept is $(0, 64)$

Interpretation of the y-intercept: The fish falls from 64 feet

The positive x-intercept is $(2, 0)$

Interpretation of this x-intercept: The fish lands in the water below after 2 seconds

The Distance vs. Time of two other fish after falling from different waterfalls are modeled by the following equations:

Trout's path: $y = -16x^2 + 81$

Perch's path: $y = -16x^2 + 13$

(B) Considering all three fish, which fell from the highest waterfall? Why?

Trout

(C) Considering all three fish, which fell for the longest period of time? Why?

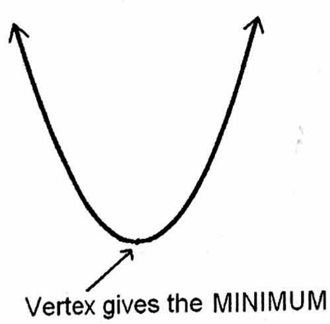
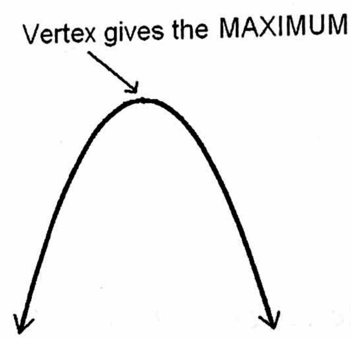
Trout

Homework: Worksheet

Lesson #63
Graphing $y = ax^2 + bx + c$

Success Criteria: I can determine if a graph will open down or up and if it will have a maximum value or a minimum value. I can determine the vertex, y-intercept and axis of symmetry without graphing.

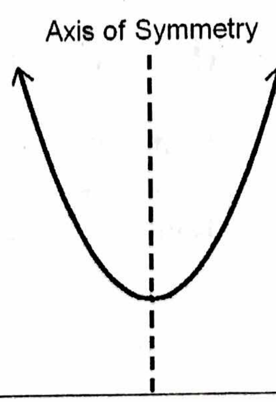
The graph of a quadratic function $y = ax^2 + bx + c$ is a parabola:

If a is POSITIVE ($a > 0$)	If a is NEGATIVE ($a < 0$) ∩
The parabola will open UPWARD Y. U	The parabola will open DOWNWARD X .
The vertex gives the MINIMUM value.	The vertex gives the MAXIMUM value.
There is NO MAXIMUM value.	There is NO MINIMUM value.
	

The <u>y-intercept</u> will be the value of c.

The <u>x-coordinate of the vertex</u> will be $x = \frac{-b}{2a}$

The <u>axis of symmetry</u> will also be $x = \frac{-b}{2a}$ (Since the vertex lies on this axis of symmetry)



Example #1: Use the quadratic function $f(x) = x^2 + 4x - 3$ for the following:

(A) Pick out the values of a, b, and c and write them below.

$a = 1$ $\quad$ $b = 4$ $\quad$ $c = -3$

(B) Will the parabola graph open upward Y or open downward I ? Why?

upward $\quad$ a positive

(C) Will the function have a minimum?

Yes

Will it have a maximum?

No

(D) Find the axis of symmetry:

$$x = \frac{-b}{2a} = \frac{-(4)}{2(1)} = \frac{-4}{2} = -2$$

(*draw it as a dotted line on the graph below)

(E) Find the x-coordinate of the vertex:

$x = -2$

(F) Find the y-coordinate of the vertex:

$y = x^2 + 4x - 3$

$= (-2)^2 + 4(-2) - 3$

$y = 4 - 8 - 3$

$y = -7$

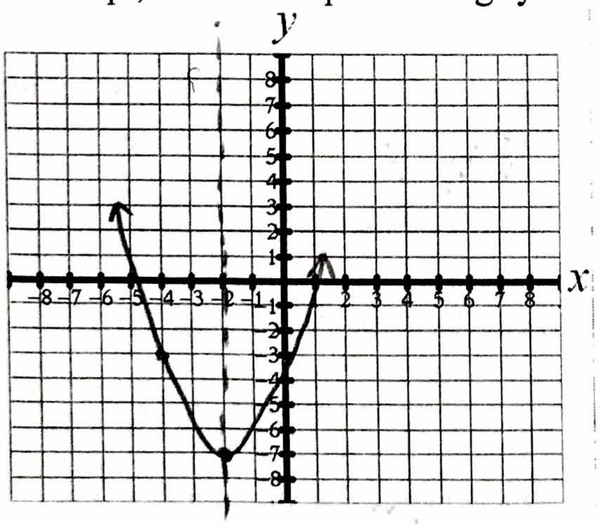
(G) Write the vertex as an ordered pair:

Vertex is $(-2, -7)$

(H) Find the y-intercept:

y-intercept is $(0, -3)$

(I) Graph the function using the vertex, the y-intercept, and a third point using symmetry.



(J) What is the minimum value of the function?

$(-2, -7)$

(K) What is the Domain and Range?

Domain: $\mathbb{R}$

Range: $y \geq -7$

Example #2: Use the quadratic function $f(x) = -\frac{1}{2}x^2 - 3x + 2$ for the following:

(A) Pick out the values of a, b, and c and write them below.

$a = -\frac{1}{2}$ $b = -3$ $c = 2$

(B) Will the parabola graph open upward Y or open downward I ? Why?

down a negative

(C) Will the function have a minimum?

No

Will it have a maximum?

Yes

(D) Find the axis of symmetry:

$x = \frac{-b}{2a} = \frac{-(-3)}{2(-\frac{1}{2})} = \frac{3}{-1} = -3$

(*draw it as a dotted line on the graph below)

(E) Find the x-coordinate of the vertex:

$x = -3$

(F) Find the y-coordinate of the vertex:

$y = -\frac{1}{2}x^2 - 3x + 2$

$= -\frac{1}{2}(-3)^2 - 3(-3) + 2$

$= -\frac{1}{2}(9) + 9 + 2$

$y = -4.5 + 9 + 2$

$y = 6.5$

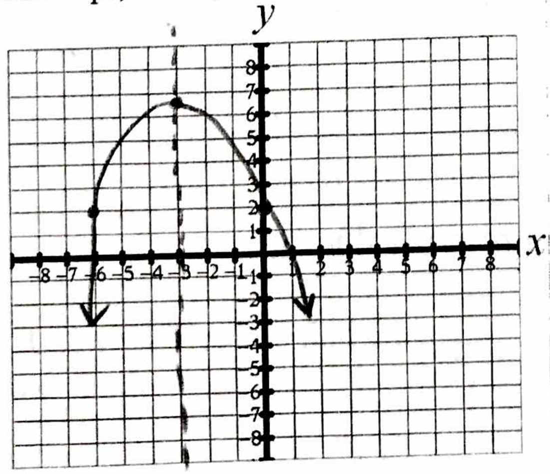
(G) Write the vertex as an ordered pair:

Vertex is $(-3, 6.5)$

(H) Find the y-intercept:

y-intercept is $(0, 2)$

(I) Graph the function using the vertex, the y-intercept, and a third point using symmetry.



(J) What is the maximum value of the function?

$(-3, 6.5)$

(K) What is the Domain and Range?

Domain: $\mathbb{R}$

Range: $y \leq 6.5$

**Note: As we did in lesson 61 and 62, we could have made a table for $f(x) = x^2 + 8x - 5$ and then plotted the points to graph the function.

However, the method we used in our first two examples, which is to use the vertex, the y-intercept, and a third point using symmetry, is a quicker method.

Success Criteria: I can find the vertex of a parabola using the calculator.

To find the vertex of a parabola using a graphing calculator:

While on the graph screen,

STEP #1: Press $\boxed{\text{2ND}}$ then press $\boxed{\text{TRACE}}$ then choose minimum OR maximum

STEP #2: Move the cursor to set a Left Bound then press $\boxed{\text{ENTER}}$,

then set a Right Bound then press $\boxed{\text{ENTER}}$,

and then press $\boxed{\text{ENTER}}$ again.

Example #3: Use a graphing calculator to find the vertex of the parabola graph for the given quadratic function, then give the maximum and the minimum value of the function.

(A) $y = \frac{2}{3}x^2 - 4x + 2$ minimum Vertex: (3 , -4) Minimum Value: -4 Maximum Value: none	(B) $f(x) = -2x^2 - 6x + 15$ Vertex: (-1.25 ,18.125) Minimum Value: none Maximum Value: 18.125

Example #4: (Group Problem) Suppose you have 200 feet of fencing and you are going to use it to enclose a rectangular space for your vegetable garden. You obviously want to have the largest area possible for the garden. What dimensions should you use?

(A) Complete the table to find several different possible combinations for the Length L and the Width W for the rectangular garden. **(You may use a calculator)**

Note: The perimeter is 200 feet, so the length and width must total half of that (or 100 feet).

Width W (in feet)	Length L (in feet)	Area $A = L \cdot W$ (in square feet)
10	90	900
20	80	1600
33	67	2211
45	55	2475
62	38	2356
70	30	2100
85	15	1275
x	$100-x$	$x(100-x)$

(B) Use the last row of your table to write an equation for the Area A when the width is x.

$A = x(100-x)$

$A = -x^2 + 100x$

(C) Graph this Area equation on your calculator.

Note: You will need to change the WINDOW values to see the vertex. Use the values in the table to get a rough idea of what your window settings need to be.

(D) Use your graph to find the maximum possible area. Record this value below, along with the width and length needed to produce this maximum area (give the correct units as well).

Maximum Area = 2500 Width = 50 Length = 50

Lesson #64

Graphing $y = a(x-h)^2 + k$

Success Criteria: I can identify what adding or subtracting and h value will do to the graph. I can identify the vertex from a table, graph or equation.

Problem #1:

(A) For each given equation, make a table of ordered pair solutions, then plot and connect the solution points to graph the equation.

Table and Graph of $y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

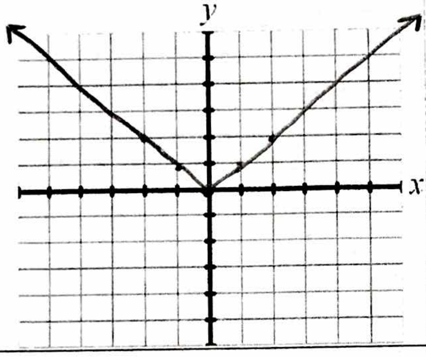
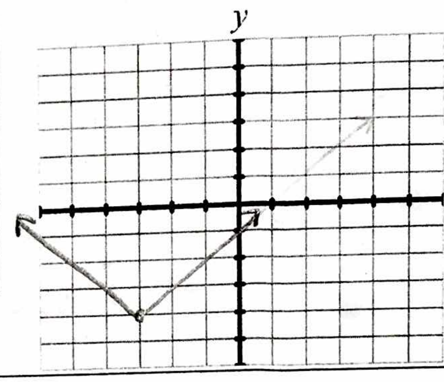


Table and Graph of $y = |x+3| - 4$

-4 -3
-3 -4

x	y
-2	-3
-1	-2
0	-1
1	0
2	1



(B) How does the graph of $y = |x+3| - 4$ compare to the graph of $y = |x|$?

left 3
down 4

Problem #2:

(A) For each given equation, make a table of ordered pair solutions, then plot and connect the solution points to graph the equation.

Table and Graph of $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

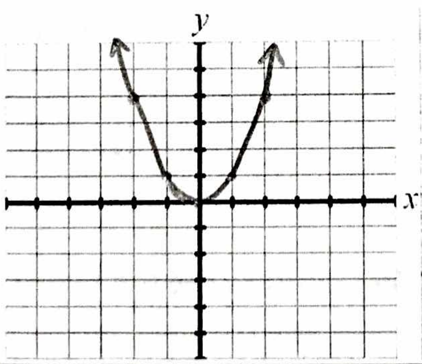
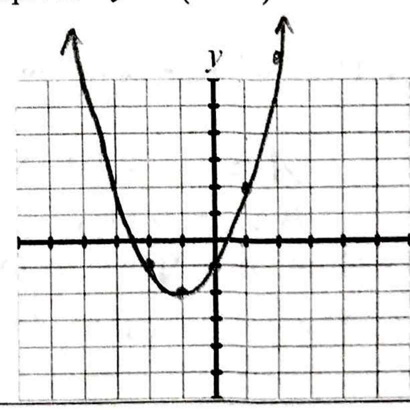


Table and Graph of $y = (x+1)^2 - 2$

x	y
-2	-1
-1	-2
0	-1
1	2
2	7



(B) What are the coordinates of the vertex of $y = (x+1)^2 - 2$? $(-1, -2)$

(C) How does the graph of $y = (x+1)^2 - 2$ compare to the graph of $y = x^2$?

left 1
down 2

Because it reveals the vertex of the parabola graph, a quadratic equation that is written in the form $y = a(x-h)^2 + k$ is said to be in **VERTEX FORM.**

The **VERTEX FORM** of a quadratic function is

$$y = a(x-h)^2 + k$$

If a is POSITIVE ($a > 0$), the parabola will open UPWARD Y.
If a is NEGATIVE ($a < 0$), the parabola will open DOWNWARD I .

As we learned previously, the larger the value of $|a|$, the narrower the parabola will be.

The VERTEX of the parabola graph will be (h, k)

To move the graph up: +k
To move the graph down: -k
To move a graph right: (x - #)
To move a graph left: (x + #)

Example #1: Give the vertex of the parabola graph, then compare the graph to $y = x^2$.

(A) $y = 3(x-1)^2 + 5$ Vertex is (1 , 5) Compare: narrower -opens up right 1, up 5	$y = -4(x+6)^2 + 9$ Vertex is (-6 , 9) Compare: narrower-open down left 6, up 9
(B) $y = (x+3)^2 - 7$ Vertex is (-3 , -7) Compare: left 3, down 7 opens up-	**(C)** $y = -8(x-5)^2$ Vertex is (5 , 0) Compare: right 5 narrower - opens down

Partner Problems #1 and #2